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Distribution

DISCUSSION OF S/N CONSIDERATIONS IN THE DESIGN OF THE REPRODUCE HEAD-PREAMPLIFIER COMBINATION

by

Philip Smaller

RB 64-20 June 1964

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AMPEX CORPORATION

RESEARCH AND ENGINEERING PUBLICATION

UC-271

ABSTRACT

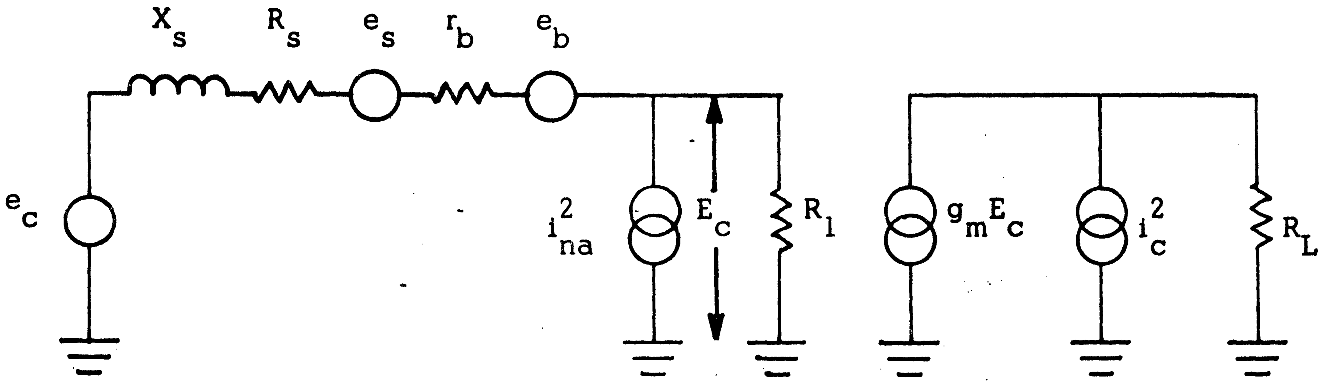
The narrowband S/N is calculated for reproduce head feeding a transistor amplifier. It is shown that optimization over a wide frequency range is impossible and the improvement in high frequency S/N can only be achieved at the expense of low frequency S/N. Similarly, it is shown that a wideband head and amplifier combination can be divided into three ranges of operation; a low frequency region in which the transistor is the main noise contributor independent of source (head) impedance; a mid-frequency range in which the head noise is the main contributor; and a high frequency range in which the transistor noise is dominant and dependent on source (head) impedance. In the mid-frequency range the S/N is shown to be proportional to $\mu'Q$ of the head material while in the high frequency range it is independent of the head materials.

PREFACE

This memo is intended to serve as an introduction to the problem of optimizing the signal-to-noise ratio of the reproduce system. As might be expected, it is the head and preamplifier which are the critical elements in the reproduce chain. Only by optimizing these two elements can one hope to achieve the best overall results.

The analysis will be limited to a reproduce head connected to a transistor amplifier. This combination was chosen because it is most common in the wideband systems. The relationships developed have their counterpart in circuits consisting of vacuum tubes, parametric amplifiers, transformer inputs, etc.

Attention will be drawn to the fact that the net result of improvements in head design, head material, or circuit improvements can only be estimated when the entire reproduce system is considered. That is, advances in any of the components will or will not have beneficial results, depending on how the component is used. The particular component of interest in this analysis is the reproduce head and the particular parameter is the head material $\mu' Q$ product.



e_c	is the signal generated by the head
X_s	is the head reactance
R_s	is the head resistance
e_s	is the head thermal noise voltage
r_b	is the base resistance
e_b	is the base resistance thermal noise voltage
i_{na}	is the emitter shot noise generator
r_e	is the emitter resistance; $R_1 = \frac{r_e}{1-\alpha}$
i_c	is the collector shot noise generator
R_L	is the load resistance

Fig. 1. Model of Transistor Amplifier

DISCUSSION

Starting with the equivalent transistor circuit[1] shown in Fig. 1, an expression for the narrowband signal-to-noise in terms of the electrical parameters has been developed. The signal-to-noise will be defined as the ratio of signal power-to-noise power per cycle bandwidth.

The signal-to-noise is found to be

$$\frac{S}{N} = \frac{e_c^2}{4KT\left[R_s + \frac{(R_s+r_b)^2}{2\beta r_\ell} + \frac{r_\ell + 2r_b}{2}\right]} \tag{1}$$

K is Boltzman's constant

T is the absolute temperature

β is transistor voltage gain

The signal, e_c, and the head impedance are functions of the head design and materials. If we assume, for the time being, that stray capacitance is negligible, then the resistance and reactance of the head can be written directly in terms of the effective permeability of the head. The effective permeability μ_ℓ is given by[2]

$$\mu_\ell = \mu'_\ell - j\mu''_\ell$$

Hence,

$$R_s = WL_o\mu''_\ell \tag{2}$$

$$X_s = WL_o\mu'_\ell \tag{3}$$

L_o is the head inductance in the absence of the core material

W is the angular frequency

The signal voltage is given by

$$e_c = \phi_T n F E \tag{4}$$

ϕ_T is the useful flux from the tape

F is the frequency

n is the head turns

E is the head efficiency

In order to write the S/N in Eq. 1 in terms of the head and transistor parameters, it is necessary to use previously developed expressions[2] for μ_ℓ and E. We will consider a head made of one material instead of the composite head. (We will not attempt to resolve here the question of whether or not the tip material or the ferrite material of the composite head controls the effective permeability of the head.)

$$\mu'_\ell = \frac{G\left(\mu'^2 + \mu''^2\right) + \mu'}{\left(G\mu' + 1\right)^2 + \left(G\mu''\right)^2} \tag{5}$$

$$\mu''_\ell = \frac{\mu''}{\left(G\mu' + 1\right)^2 + \left(G\mu''\right)^2} \tag{6}$$

$\mu = \mu' - j\mu''$ is the permeability of the head material

$$G = \frac{A_c}{A_g}\,\frac{\ell_g}{\ell_c} \tag{7}$$

A_c is the core area

A_g is the gap area

ℓ_g is the gap length

ℓ_c is the core length

Similarly, the efficiency is given by

$$E^2 = G^2 \left[\frac{\mu'^2 + \mu''^2}{(G\mu'+1)^2 + (G\mu'')^2} \right] \tag{8}$$

The effective Q of the head, Q_e, is that which is measured on a bridge.

$$Q_\ell = \frac{X_s}{R_s} = \frac{W L_o \mu'_\ell}{W L_o \mu''_\ell} = \frac{\mu'_\ell}{\mu''_\ell} \tag{9}$$

From Eq. (5), (6), and (9)

$$Q_\ell = \frac{G(\mu'^2 + \mu''^2) + \mu'}{\mu''} \tag{10}$$

For the ferrite materials of interest it can be shown that $\mu' \gg \mu''$ over the frequency range of concern. Hence, Eq. (10) becomes

$$Q_\ell \approx \frac{\mu'}{\mu''}\left(G\mu'+1\right) = Q_m\left(G\mu'+1\right) \tag{11}$$

where Q_m is the magnetic Q of the head materials. If we now make the approximation that for the small gap heads we are presently using (ℓ_g = 25 μ-inches) that $G\mu' \ll 1$, then Eq. (11) becomes

$$Q_\ell \approx Q_m \tag{12}$$

Similarly, the efficiency of the head can be written as

$$E^2 \approx G^2 \mu'^2 \tag{13}$$

Equation (12) indicates how improvements in the Q of the head materials result in an increase in the effective Q of the head.

Returning to Eq. (1) and expanding

$$\frac{S}{N} = \frac{e_c^2}{4KT\left[R_s + \frac{r_b^2 + R_s^2 + 2r_b R_s}{2\beta r_\ell} + \frac{X_s^2}{2\beta r_\ell} + \frac{r_\ell + 2r_b}{2}\right]} \tag{14}$$

$$= \frac{e_c^2}{4KT\left[R_s + \frac{1}{2\beta r_\ell}\left(R_s^2 + X_s^2\right) + \frac{2r_b R_s}{2\beta r_\ell} + \frac{r_b}{2\beta r_\ell} + \frac{r_\ell + 2r_b}{2}\right]} \tag{15}$$

Choosing the transistor parameters such that

$$\frac{r_b}{2\beta r_\ell} << 1$$

Equation (15) reduces to

$$\frac{S}{N} \approx \frac{e_c^2}{4KT\left[R_s + \frac{R_s^2 + X_s^2}{2\beta r_\ell} + \frac{r_\ell + 2r_b}{2}\right]} \tag{16}$$

substituting $\frac{X_s}{R_s} = Q_\ell$

$$\frac{S}{N} \approx \frac{e_c^2}{4KT\left(\frac{X_s}{Q_\ell} + \frac{X_s^2}{2\beta r_\ell}\left(\frac{1 + Q_\ell^2}{Q_\ell^2}\right) + \frac{r_\ell + 2r_b}{2}\right]} \tag{17}$$

For $Q_e >> 1$ and $e_c^2 \approx \phi_T^2 n^2 F^2 G^2 \mu'^2$

$$\frac{S}{N} \approx \frac{\phi_T^2 n^2 F^2 G^2 \mu'^2}{4KT\left(\frac{X_s}{Q_\ell} + \frac{X_s^2}{2\beta r_\ell} + \frac{r_\ell + 2r_b}{2}\right)} \tag{18}$$

$$\approx \frac{B n^2 F^2}{\frac{X_s}{Q_\ell} + \frac{X_s^2}{2\beta r_\ell} + \frac{r_\ell + 2r_b}{2}} \qquad \text{where} \qquad B = \frac{\phi_T^2 G^2 \mu'^2}{4KT}$$

Equation (18) is a very useful expression of the dependency of S/N head turns, head geometry, material permeability or transistor parameters. The denominator of Eq. (18) contains three terms. It is the behavior of these terms which determines the performance of the head-preamplifier combination. We can now describe three major regions of operation in the frequency domain.

1) At very low frequencies (or small values of n) the last term in the denominator will dominate and the S/N will be

$$S/N \approx \frac{\phi_T^2 F^2 n^2 G^2 \mu'^2}{4KT\left(\frac{r_e + 2r_b}{2}\right)} \tag{19}$$

If the d-c resistance of the head is significant, then it also may be added to the denominator. The S/N is seen to be proportional to $n^2 \mu'^2$. For this reason, high permeability material would be helpful. The number of turns which can be wound is limited by high frequency considerations and will be discussed later. The emitter and base resistance should be low. This requirement of low r_e will be shown to be in conflict with high frequency requirements.

2) The second region of operation will usually be the case where the frequency is increased and the first term of Eq. (18) will dominate. When that occurs, the S/N is given by

$$S/N \approx \frac{\phi_T^2 F^2 n^2 G^2 \mu'^2}{4KT\left(\frac{X_s}{Q_L}\right)} \tag{20}$$

If we write

$$X_s = WL_0\mu' = 2\pi F A n^2 \mu' \tag{21}$$

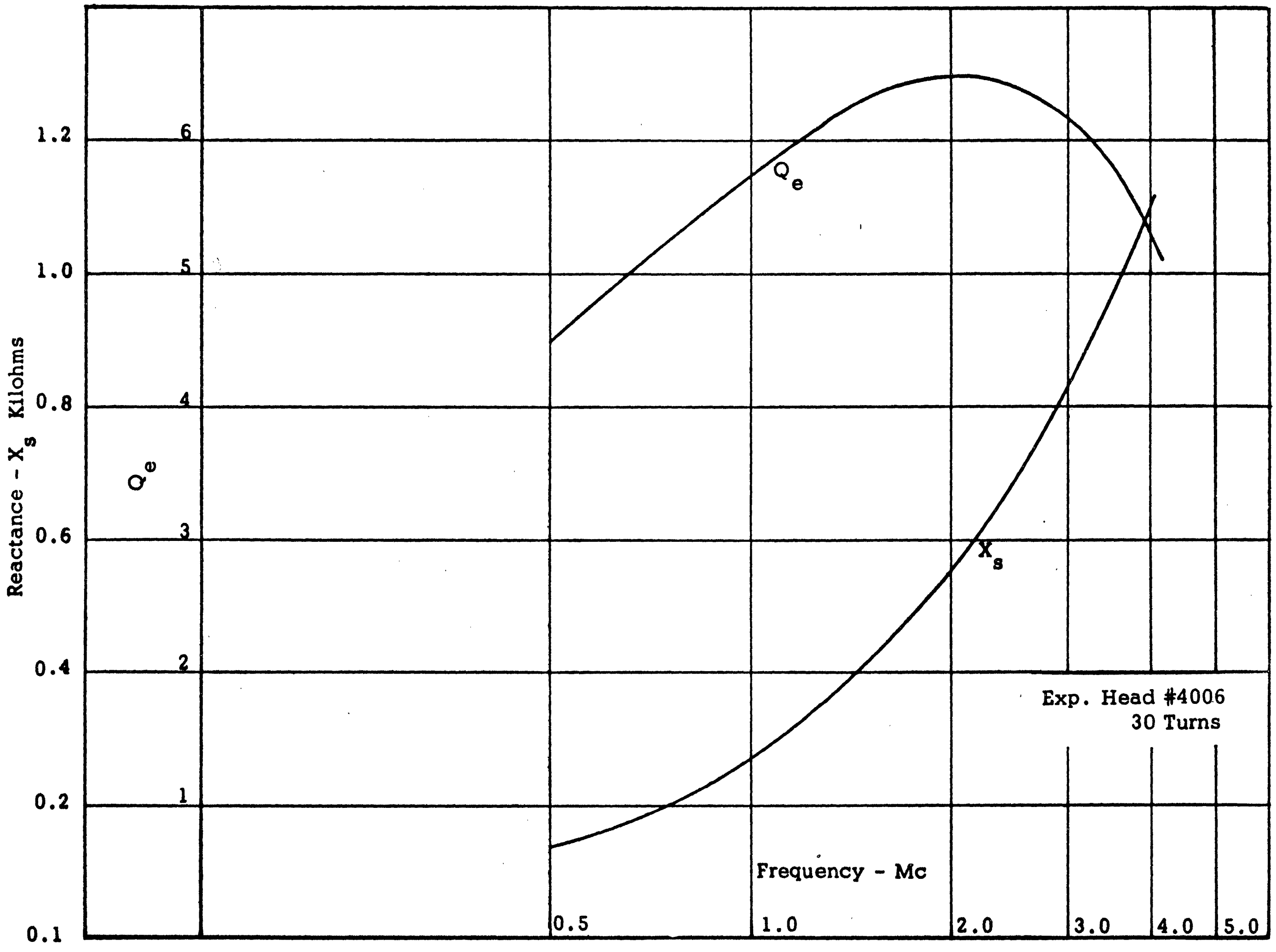


Fig. 2. Reactance and Q_2 of FR-1600 Mark IV Experimental Head as a Function of Frequency

where A is a geometrical factor for the head, then Eq. (20) becomes

$$\frac{S}{N} \approx \frac{\phi_T^2 F G^2}{8\pi K T}\left(\mu' Q_\ell\right) \tag{22}$$

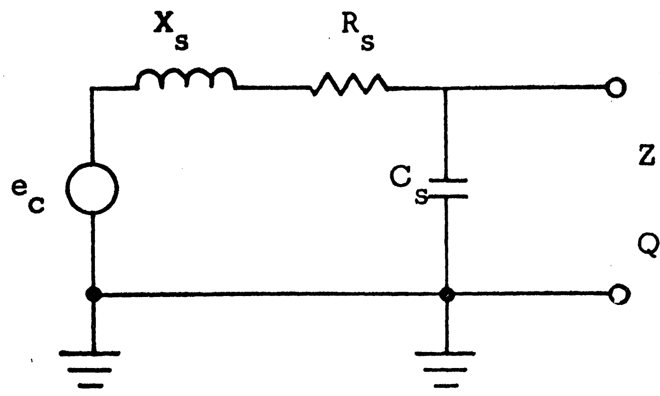
It is in this region of operation that improvements in the $\mu' Q_e$ of the head will result in improvement in the S/N. Effort should be made to make this region of operation as large as possible by keeping the other two terms in the denominator of Eq. (18) comparitively small.

3) At very high frequencies (large values of n, or small values of $2\beta r_\ell$), it is the middle term of Eq. (18) which dominates. The S/N is then

$$\frac{S}{N} \approx \frac{\phi_T^2 F^2 n^2 G^2 \mu'^2}{4KT\left(\dfrac{(2\pi)^2 F^2 A^2 n^4 \mu'^2}{2\beta r_\ell}\right)} = \phi_T^2 G^2 \left(\frac{\beta r_\ell}{16\pi^2 A^2 n^2}\right) \tag{23}$$

To maximize the S/N in this region, it is advisable to use large values of r_ℓ and small values of n. This requirement is in conflict with the low frequency considerations; hence, a trade-off is required between high frequency and low frequency S/N.

The concepts expressed thus far can be used to calculate the relative S/N at the output of the transistor amplifier used in conjunction with the FR-1600 head. Measured value of Q_e and X_s for a proposed FR-1600 head is shown in Fig. 2. These values of Q_e and X_s can be substituted into Eq. (18) and the S/N calculated



$$Z' = R_s' + j\,X_s'$$

$$Q_e' = \frac{X_s'}{R_s'}$$

C_s is the stray capacitance

Z' is the impedance looking back from the transistor input terminals

Fig. 3. Head Circuit with Stray Capacitance

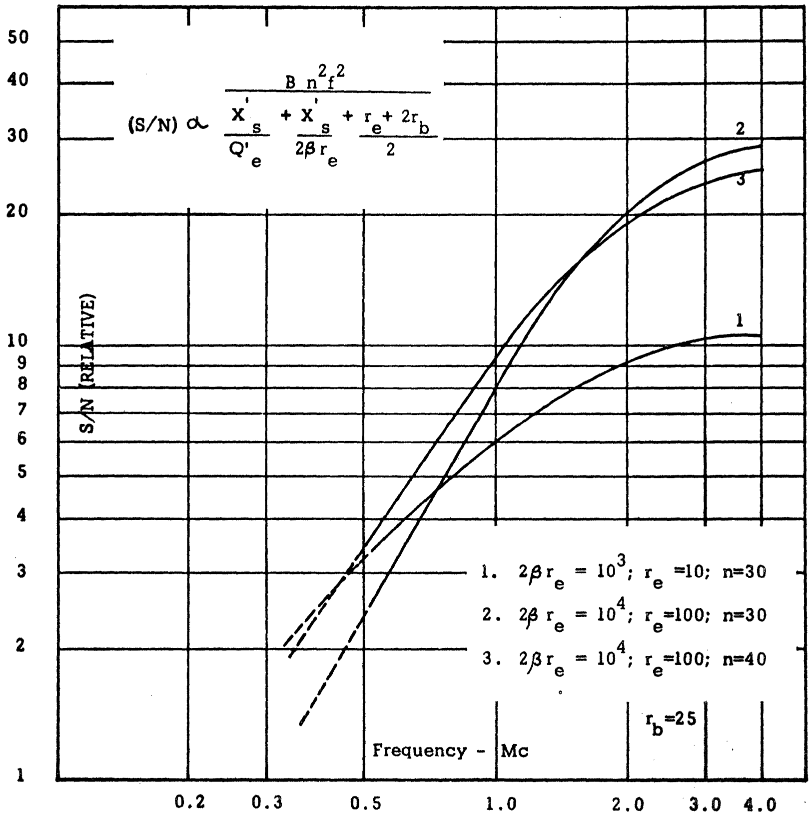


Fig. 4. Narrow Band Signal-to-Noise Rates of Input Circuit as a Function of Frequency

as a function of frequency. However, more accurate results are obtained if we assume a value of stray capacitance across the head as shown in Fig. 3. Then the term Q_e in Eq. (18) is replaced by Q'_e and the term X_s is replaced by X'_s. Eq. (18) is valid for values of $Q'_e \gg 1$, otherwise Eq. (17) must be used. The value of capacitance chosen was 15 μμF, which gave a resonant frequency of 6 MC. The calculation of S/N was then made and the results plotted in Fig. 4. Curves were plotted for different values of $2\beta r_\ell$, r_ℓ, and n.

From Fig. 4, Curve 1, we can see that low values of $2\beta r_\ell$ result in low S/N at high frequencies, but in a higher S/N at low frequencies. This was explained previously to be due to the $X_s^2/2\beta r_\ell$ term in Eq. (18). To raise the high frequency S/N, it is necessary to increase r_ℓ. Curve 2 shows the S/N when r_ℓ has been increased from $r_\ell = 10$ to $r_\ell = 100$. The S/N at high frequencies has been increased at the expense of the low frequency S/N. Curve 3 illustrates how a change in head turns will also modify the S/N behavior of the system.

It can be shown by reference to Eq. (18) that, for the case of Curve 1, which is representative of the present circuit parameters of the FR-1600 pre-amplifier, increases in $\mu' Q_e$ will not improve the S/N at high or mid-range frequencies. This is because the middle term of Eq. (18), $X_s^2/2\beta r_\ell$, is dominant over the entire frequency range with the exception of very low frequencies. Only by increasing $2\beta r_\ell$, as shown by Curve 2, does the $X_s^2/2\beta r_\ell$ term decrease to the degree that the first term, X_s/Q_e, becomes most significant, and thereby allows for increasing S/N with increasing $\mu' Q_e$, in accordance with 2).

CONCLUSIONS AND RECOMMENDATIONS

I. As the bandwidth increases, it becomes impossible to optimize the S/N over the entire band. Improvements in high frequency S/N can only be made at the expense of low frequency S/N.

II As one changes the bandwidth of a multiple speed machine, it is probable that one will want to change transistor and head parameters in much the same way as equalization is now changed.

III Improvements in head design and materials will not result in increased performance unless the preamplifier is specifically designed to take advantage of the changes.

IV Because of the critical nature of the head and preamplifier it is becoming increasingly difficult for a product development group to become knowledgeable of recent advances in transistors and other active amplifiers, head design and head materials, and to weigh the importance of these advances. This is made more difficult by the fact that no general design criteria for the head-preamplifier is in existence. Similarly, it is difficult for the product planning group to outline specifications without knowing the implications and trade-offs implicit in these specifications. For these and other reasons, it appears wise to establish an off-line group which bears continuing responsibility for the head-preamplifier design problem. Such a group would serve as a focal point for the diverse information and talents which are necessary to achieve optimum results. Once the design parameters for our present heads and amplifiers have been established, it will be a relatively easy task to evaluate the benefits of new devices and circuits, for example, microelectronic circuits. Continued success in the magnetic recording field requires that we treat the reproduced signal as carefully and intelligently as possible. A unified group applying themselves to doing this task is required.

Acknowledgement

The assistance of Allen Grace in suggesting the particular transistor model and in deriving the basic S/N equation and to Nelson Yew of the Head Development Group for data on the FR-1600 head is gratefully acknowledged.

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1. Linvill & Gibbons, "Transistors and Active Circuits," McGraw Hill, p 394

2. P. Smaller, "Design of Reproduce Heads for Maximum S/N," RB 63-2