



REVISION NOTICE

This publication replaces previous descriptions of "Solution of Quartic Equations," program C2-35.0. Descriptions have been clarified and program references changed to current nomenclature.

FUNCTION

"Solution of Quartic Equations" is used to determine all roots (real and imaginary) of the 4th Order Polynomial

$$a_0z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0$$

INPUT

1. Store the program in Lo.
2. Arrange  $a_0 = 1$ .
3. Store data in Lo + 0600.
4. Store flags, counters, and constants as indicated on "DATA LOAD SHEET," page 4.
5. Until the program is revised it will be necessary to store zeros in L0 + 0605 to Lo + 0635, inclusive.

OUTPUT

Set tab stops for a 4 column printout. The printout will be in the form  $A = B_i$ .

## SOLUTION OF QUARTIC EQUATIONS

### PROCEDURE

Transfer to beginning of program and start with the MANUAL INPUT lever on the typewriter depressed. The light on the typewriter will then come on and the operator can feed the prepared data through the reader by lifting the typewriter MANUAL INPUT lever. The computer will store the data, make a carriage return, and then start to compute the roots.

After the roots have been printed out it will be necessary to depress the START button on the computer and the following operations will be executed:

1. The given coefficients will be printed out.
2. Check coefficients will be computed from the roots.
3. The check coefficients will be printed out.

If the coefficients do not compare closely it is necessary to change the flag in Lo + 0640 to a higher number and repeat the entire process.

### EXAMPLE I

In the event a program should stop due to an excessive exponent it is necessary to scale the coefficients of the equation.

Use the following method:

Given

$$f(x) = x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \dots + a_n$$

Substitute

$$10^m y = x$$

Then

$$f(y) = y_n + \frac{a_1 y^{n-1}}{10^m} + \frac{a_2 y^{n-2}}{10^{2m}} + \frac{a_3 y^{n-3}}{10^{3m}} + \dots + \frac{a_n}{10^{nm}}$$

SOLUTION OF QUARTIC EQUATIONS

EXAMPLE I (Cont.)

In terms of its roots  $f(y)$  can be expressed

$$f(y) = (y - r_1) (y - r_2) \dots (y - r_n)$$

but,

$$y = \frac{x}{10^m}$$

and

$$\frac{1}{10^m} f(x) = \frac{1}{10^m} [(x - 10^m r_1)(x - 10^m r_2)(x - 10^m r_3) \dots (x - 10^m r_n)]$$

From the above it can be easily seen that the roots of  $f(y)$  are  $1/10^m$  of the roots of  $f(x)$ .

EXAMPLE II

Cubic equations have been set up for this program to solve by multiplying the cubic by  $(x - r)$ , where  $(x - r)$  is a factor that is selected at random.

Since  $f(x) = (x - r) f(x)$ , it can be seen that by eliminating  $(x - r)$  from the roots of the resultant quartic we have the roots of the cubic.

NOTE

This entire subroutine is coded in floating point. The Floating Point Interpretive System 1, program H1-24.0, must be in memory and the first two instructions of this subroutine must be addressed to enter it.

SOLUTION OF QUARTIC EQUATIONS

DATA LOAD SHEET

NOTES	p	±	q	LOCATION	h	±	NUMBER	STOP	CGE	RET
a <sub>0</sub>				Lo+ 600	/			/		
a <sub>1</sub>					/			/	X	
a <sub>2</sub>				to	/			/		
a <sub>3</sub>					/			/	X	
a <sub>4</sub>				Lo+ 604	/			/		
					/	-	0 0 0 0 0 0 0 0	/	X	
				Lo+ 605	/			/		
				to	/			/	X	
				Lo+ 635	/			/		
					/	-	0 0 0 0 0 0 0 0	/	X	
F = FLAG	F	+	0 0	Lo+ 636	/			4	/	
C = COUNTERS	C				/			1	/	X
K = CONSTANTS	K				/			1	/	
	F				/			2	/	X
	F				/			n	/	
	K				/			4	/	X
	K				/			3	/	
	K				/			2	/	X
	F				/			1	/	
	C				/			1	/	X
	K				/			1	/	
					/	-	0 0 0 0 0 0 0 0	/	X	
	C			Lo+ 652	/			1	/	
					/	-	0 0 0 0 0 0 0 0	/	X	
	K			Lo+ 656	/	-	0 0 0 0 0 0 0 2	/		
					/	-	0 0 0 0 0 0 0 0	/	X	
					/				/	
					/				/	X
					/				/	
					/				/	X
					/				/	
					/				/	X
					/				/	

Store zeros

Store number of iterations (Suggest 10)

C2-35.0

SOLUTION OF QUARTIC EQUATIONS

Baudlauf

xr0000'xu0000'xi0000'b0637'e0638'b0600'e0607'  
b0601'e0608'b0602'e0609'b0603'  
c0610'b0604'e0611'u0019'xz0000'  
xz0000'xz0000'e0056'y0028'e0057'  
y0031'e0058'y0033'e0059'y0036'  
p0615'm0000'e0647'p0626'm0000'  
c0648'b0000's0647's0648'e0000'  
b0636's0638't0060'b0638'a0637'  
c0638'e0028'xi0001'y0028'e0031'  
xi0001'y0031'e0033'xi0001'y0033'  
e0036'xi0001'y0036'u0027'z0617'  
z0616'z0607'z0618'u0061'b0621'  
c0612'b0622'e0613'b0637'e0638'  
e0140'y0112'e0141'y0115'e0142'  
y0117'e0143'y0120'p0615'm0000'  
c0649'p0626'm0000'e0650'b0000'  
s0649's0650'e0000'b0639's0638'  
t0144'b0638'a0637'e0638'e0112'  
xi0001'y0112'e0115'xi0001'y0115'  
e0117'xi0001'y0117'e0120'xi0001'  
y0120'u0111'z0628'z0627'z0618'  
z0629'b0637'e0638'b0630'e0633'  
b0631'h0623'e0634'b0626'xt0000'  
xu0000'm0630'e0651'b0615'xt0000'  
xu0000'm0631'a0651'h0632'e0624'  
u0200'p0624'm0633'e0651'  
p0631'm0631's0651'e0614'p0613'  
m0630'e0651'p0612'm0631's0651'd0614'  
c0625'p0613'm0631'e0651'p0612'  
m0624's0651'd0614'e0635'b0615'  
a0625'e0615'b0626's0635'e0626'  
b0640's0652't0236'b0652'a0637'  
c0652'u0003'b0541'h0700'h0701'h0702'h0703'  
h0704'h0705'h0706'h0707'e0733'  
p0615'm0615'e0642'p0626'm0637'  
xu0000'm0641'e0653'b0642's0653'  
h0658't0322'b0700'xt0000't0302'  
e0320'y0313'xi0002'y0318'u0306'  
e0321'y0313'xi0002'y0318'b0658'

xr0000'e0659'b0615'xy0000'  
a0659'd0643'c0000'b0615'xy0000'  
s0659'd0643'c0000'u0360'z0700'z0704'  
b0700'xt0000't0332'e0358'y0346'  
y0355'x10001'y0342'y0353'u0338'  
e0359'y0346'y0355'x10001'y0342'  
y0353'b0658'xb0000'xr0000'd0643'  
c0000'b0615'xy0000'd0643'c0000'  
e0342'x10002'y0354'e0346'x10002'  
y0356'b0000'c0000'b0000'c0000'  
u0360'z0700'z0704'b0619'c0615'  
b0620'u0400'c0626'b0644's0645'  
t0408'b0645'a0646'c0645'u0246'  
b0700'xp0000'b0701'xp0000'  
b0702'xp0000'b0703'xy0000'  
h0703'xp0000'b0704'xp0000'  
b0705'xp0000'b0706'xp0000'  
b0707'xy0000'h0707'xp0000'xz0000'  
b0601'xp0000'b0602'xp0000'  
b0603'xp0000'b0604'xp0000'  
b0700'a0702'h0708'a0704'  
a0706'h0709's0708'c0710'p0710'  
m0708'c0711'p0707'm0707'c0712'  
p0707'm0704'c0713'p0707'm0706'  
c0714'p0704'm0706'a0712'a0713'  
s0714'c0715'p0715'm0708'c0716'  
p0703'm0703'c0717'p0703'm0700'  
c0718'p0703'm0702'c0719'p0700'  
m0702'a0717'a0718's0719'c0720'  
p0720'm0710'c0721'b0715'a0720'  
a0711'c0722'b0716'a0721'  
c0724'p0720'm0715'c0723'b0709'  
xy0000'xp0000'b0722'xp0000'  
b0724'xy0000'xp0000'b0723'  
xp0000'xe0000'xz0000'.0000000'