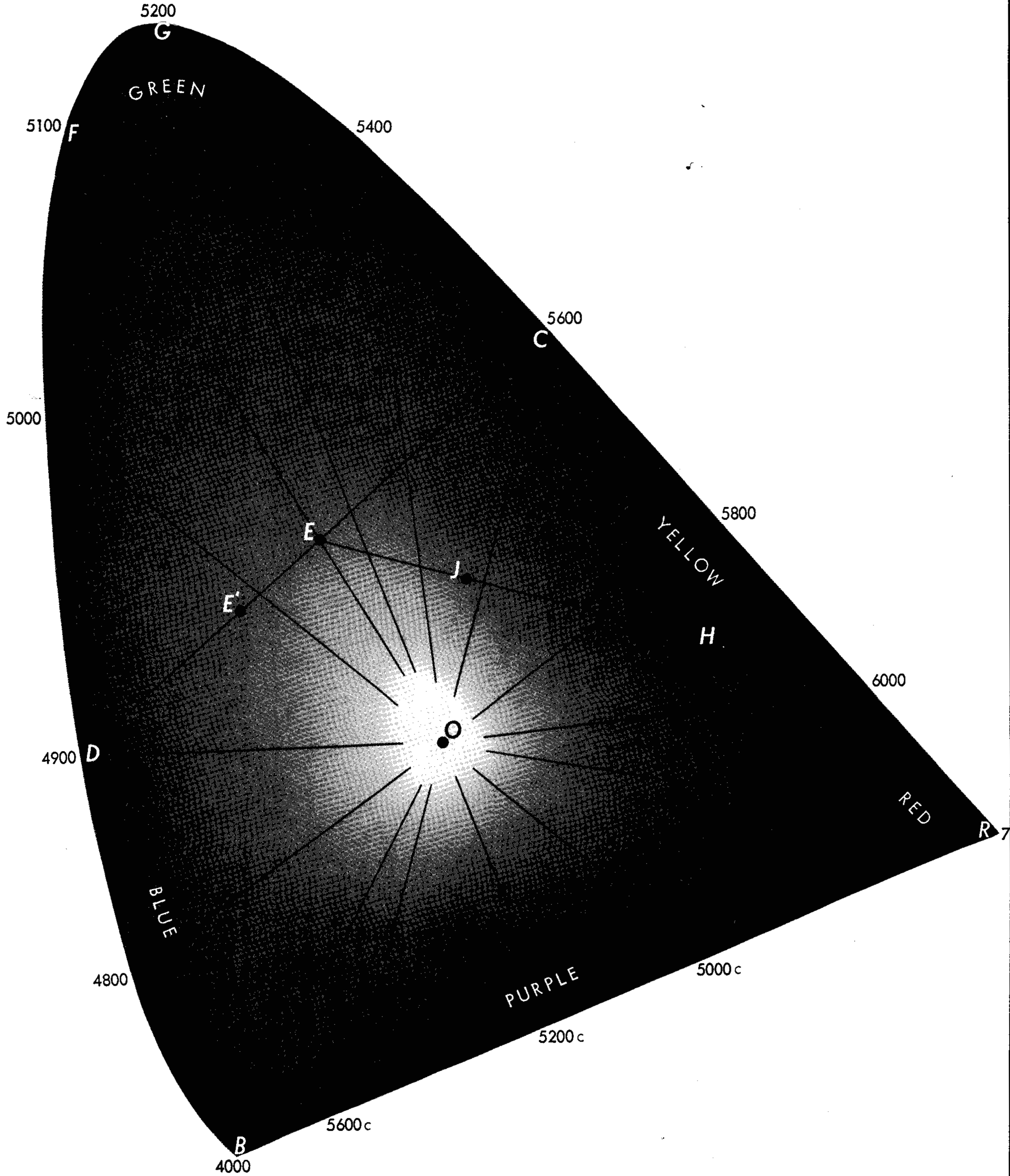


Figure 1 **The I.C.I. chromaticity diagram.**

This standard color chart, constructed from empirical data, can be used to determine the degree of color saturation and the hue seen by the eye when two or more beams of light are mixed. A combination of pure blue light (D) and a pure yellow-green (C) are thus seen as a very pale blue-green (E). The peripheral coordinates are in angstrom units.

M. M. Woolfson

Some New Aspects of Color Perception

Abstract: A mathematical analysis is made of Land's recent experiments which showed that fully colored pictures can be produced by a two-color projection system. Although Land's results apparently had been at variance with the classical theories of color perception, it has now been found possible to explain the experiments within the framework of those theories and in conjunction with well-known phenomena in the field of experimental psychology. The results are interpreted in terms of a mechanism of color transformation.

Introduction

Recent publications by Land[1-3] described some remarkable and rather unexpected results that he obtained with a system of color projection. His experimental arrangement was similar to the one by which Maxwell, in 1855, may be considered to have produced the first color photograph. Maxwell, who had at his disposal only black-and-white photographic plates, took three photographs of the same scene, each through a different filter—one red, one green and one blue. Positives prepared from these plates were projected simultaneously, each through its corresponding filter, from three different projectors, and the precisely overlapped pictures reproduced the original scene in full color.

In repeating Maxwell's work, Land made the astounding observation that even when one of the projectors was switched off, say the one with the blue filter, the full range of spectral colors *including blue* could still be seen.* The red and green light beams from the remaining projectors seemed able in some way to combine on the screen to give the sensation of blue. This result apparently clashes violently with classical, accepted theories of color and color perception, and Land suggested as an outcome of his work that these theories were no longer tenable.

The purpose of this paper is to show that Land's results can be explained within the framework of our present knowledge of color and its perception. As an interesting by-product of this investigation, there emerges a clearer picture of the mechanism by which the eye behaves as a receiver and interpreter of color information. Recourse is made to some previously known psycho-physiological aspects of color vision and some new points are suggested in this field.

*Somewhat similar phenomena had been observed by W. F. Fox and W. F. Hickey in 1914, as noted by E. J. Wall, *History of Three-Color Photography*, 1925, pp. 596-597.

The earlier sections of the paper will be devoted to a brief resume of the accepted theory of color and its perception. It will then be shown that Land's results are not only consonant with the theory but are even corroborative to some extent.

The perception of color and color mixtures

It is almost three hundred years since Newton, with his famed prism experiments, first showed the chromatic nature of white light and provided the key to the understanding of the phenomenon of color and its perception. Newton derived some good empirical rules to describe the chromatic effects of adding variously colored beams of light and, a little over a century ago, work by Maxwell, Young and Helmholtz put these rules on a firm mathematical foundation. The most modern interpretation of these results is illustrated in Fig. 1, the chromaticity diagram produced by the International Commission on Illumination.

Around the curved portion of the periphery of the diagram are marked the wavelengths from 4000 to 7000 A, the limits of the visible spectrum. The effect of adding a pair of pure spectral colors, as beams of light for example, may easily be found from the diagram. We shall take as our example the addition of two monochromatic beams of light of equal intensities and wavelengths 4900 A and 5600 A, corresponding to the points C and D on the diagram. The resultant chromatic effect would be represented by the point E at the midpoint of CD. The hue seen by the eye would be equivalent to the spectral color of wavelength 5100 A but, since the point E is well away from the boundary towards the point O, which corresponds to pure white, the color will be pale and unsaturated. The degree of saturation can be defined by the ratio OE/OF which in this case, expressed as a percentage, is 33%. The resultant of the color addition may be fully described as having hue equivalent to 5100 A and saturation 33%, which completely defines its position within the chromaticity diagram.

If the relative intensities of the beams of wavelengths 4900 A and 5600 A had been 2:1, then the resultant would be at E', where $DE':E'C=1:2$.

It will be seen from Fig. 1 that certain pairs of wavelengths, combined with the correct relative intensities, may give the appearance of a pure white; a blue beam of light of wavelength 4800 A and a yellow beam of wavelength 5800 A and approximately the same intensity will combine in this way. It should be noted that although a physical measuring instrument, such as a spectroscope, would easily distinguish between ordinary white light and a mixture of blue and yellow (it would see one as a complete spectrum and the other as two individual wavelengths) the eye would be unable to distinguish the two.

The effects of adding more than two colors may be found by the continuous application of the principle already described. For example, the resultant of adding a third colored beam of light, whose chromaticity was represented by the unsaturated point H, to the two previously considered and with the same intensity, will be the point J, where $EJ:JH=1:2$. The point E has twice the weight of point H since it is the resultant of two beams of light of equal intensity to the beam corresponding to H.

It is fairly clear from Fig. 1 that three saturated colors, with spectral wavelengths 4000 A, 7000 A and somewhere in the region 4900 to 5800 A, combined in various proportions, are capable of giving all possible hues. In general, one may produce a resultant anywhere within the triangle defined by the points corresponding to the three colors. In particular, if the three wavelengths are represented by B, G and R, then practically any point within the chromaticity diagram may be simulated.

The straight line joining 4000 A and 7000 A represents mixtures of red and blue giving rise to shades of purple, a nonspectral color. However, to describe purple in spectral terms, the complementary spectral wavelength, usually corresponding to a green, is given with the appended letter C.

The Young-Helmholtz theory of color vision

The Young-Helmholtz theory is a model of the way in which the eye distinguishes color. The theory provides a basis for understanding the information contained in the chromaticity diagram, Fig. 1.

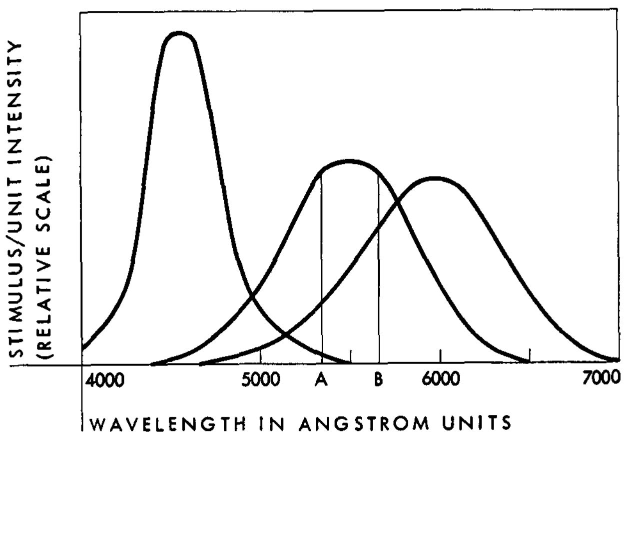
The fundamental hypothesis of the Y-H theory is that the eye contains three types of color sensor, each stimulated by all the wavelengths of the visible spectrum, but each type giving a maximum response for some different color. The type of sensor is described by its peak-response color, i.e., we have red, green and blue sensors.[4] Figure 2 is a diagrammatic representation of the stimulus-wavelength relationship for each type of sensor for monochromatic lights of unit intensity. The curve for red is sometimes shown with a small additional maximum at about 4300 A. If light of a certain wavelength, say 6000 A, falls on a portion of the retina, then the color sensors in that region are stimulated in the proportions given by the ordinate at 6000 A. Conversely, these proportions of stimuli will be interpreted by the visual mechanism as being due to the spectral wavelength 6000 A, and the sensation of color is due to the appreciation and interpretation of the proportions of the three types of stimulus.

We shall now examine some characteristics of this mechanism for recognizing color. There are, for example, two greens—one a blue-green represented by the point A in Fig. 2 and the other a yellow-green represented by the point B, which give the same green-sensor stimulus. However, they are distinguished by the proportions of the lesser blue and red stimuli. Another feature of the mechanism is that white light may be assumed to affect the three sensors equally. The converse relationship is also true—any combination of colored lights which affects the three sensors equally will be indistinguishable to the eye from white light. For example, yellow and blue lights of equal intensity and wavelengths 4800 A and 5800 A, respectively, will affect the sensors by the amounts shown in Fig. 3. The sum of the stimuli, produced by the combination of lights, is seen to be equivalent to that produced by white light, and the two colors are therefore complementary.

We are now in a position to understand more fully the terms *hue* (as distinct from spectral color) and *saturation*. It was seen in Fig. 1 that the addition of two monochromatic beams of light of equal intensities and wavelengths 4900 A and 5600 A will give a resultant with hue 5100 A and saturation 33%. The effects of adding the two sets of stimuli are shown in Fig. 4, but an examination of Fig. 2 reveals no wavelength giving the same relative stimuli as the resultant. However, when a constant quantity is subtracted from each stimulus, the remainders have the same relative proportions as do the stimuli for a wavelength of 5100 A. The amounts subtracted from each stimulus correspond to a white background and the saturation of the resultant hue can be measured by the ratio

$$\frac{\text{sum of stimuli above white background}}{\text{sum of stimuli}}.$$

Figure 2 **The stimulus-wavelength relationships for the Young-Helmholtz color sensors.**



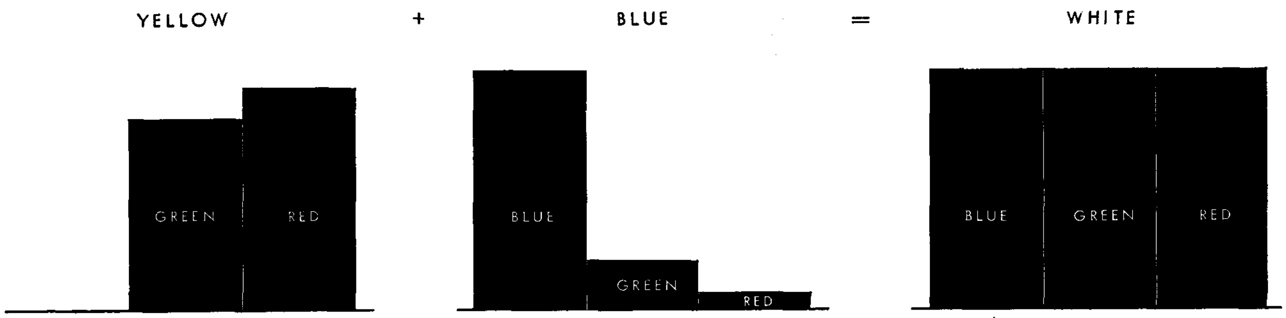


Figure 3 **Complementary colors are those which combine to give equal stimuli to the three color sensors.**

Figure 4 **The addition of pure spectral colors to give unsaturated colors.**

The mechanism suggested by the Y-H theory is in concordance with the experimentally observed facts of color vision and gives a neat explanation of these facts. While the actual forms of the stimulus-wavelength relationships are not known with certainty, the curves shown in Fig. 2 are in reasonable agreement with those suggested by most workers in this field.

The Land experiments

The experiments carried out by Land were far more comprehensive than just the elimination of one of the colors of Maxwell's projection arrangement. Land did most of his work with color-separated positives taken through a red and green filter, respectively. Other pairs of color filters would have done equally as well as long as the bands of wavelength transmitted by them were sufficiently different. The positives prepared by photography through the red and green filters were called the *long* and *short records,* respectively, the terms *long* and *short* describing the band of wavelengths transmitted by the filters. In his more controlled experiments, Land projected the records with monochromatic light of two different wavelengths, the long record with the longer of the two. We shall now summarize the most important results obtained by Land which any satisfactory theory would be called upon to explain.

1. When the two projection wavelengths are sufficiently different, an almost full-color picture results. The colors which do not appear for various combinations of projection wavelengths are shown by Land's empirical data in Fig. 5. The so-called "achromatic region" near the diagonal occurs when the projection wavelengths are insufficiently separated; in fact, the picture will not be truly achromatic but will show only color mixtures of the projection wavelengths.

The region under the diagonal corresponds to using the shorter projection wavelength with the long record. This gives a color-reversal effect—reds tend to appear as blue-greens, blues as orange-reds, and so on. There is another color-reversal area where two short wavelengths are used for projection in the usual way, that is to say, the long record with the longer wavelength.

2. One of the projections can be made with white light. If the other projection wavelength is greater than about 5880 A, then the white light must be used for the short record; otherwise it is used for the long record. Land found that when red and white were used to project the long and short records, then the projected picture could be photographed with color film, giving a result similar to that seen by the eye. However, with two monochromatic projections the color film would record only the projection colors and their Newtonian mixtures.

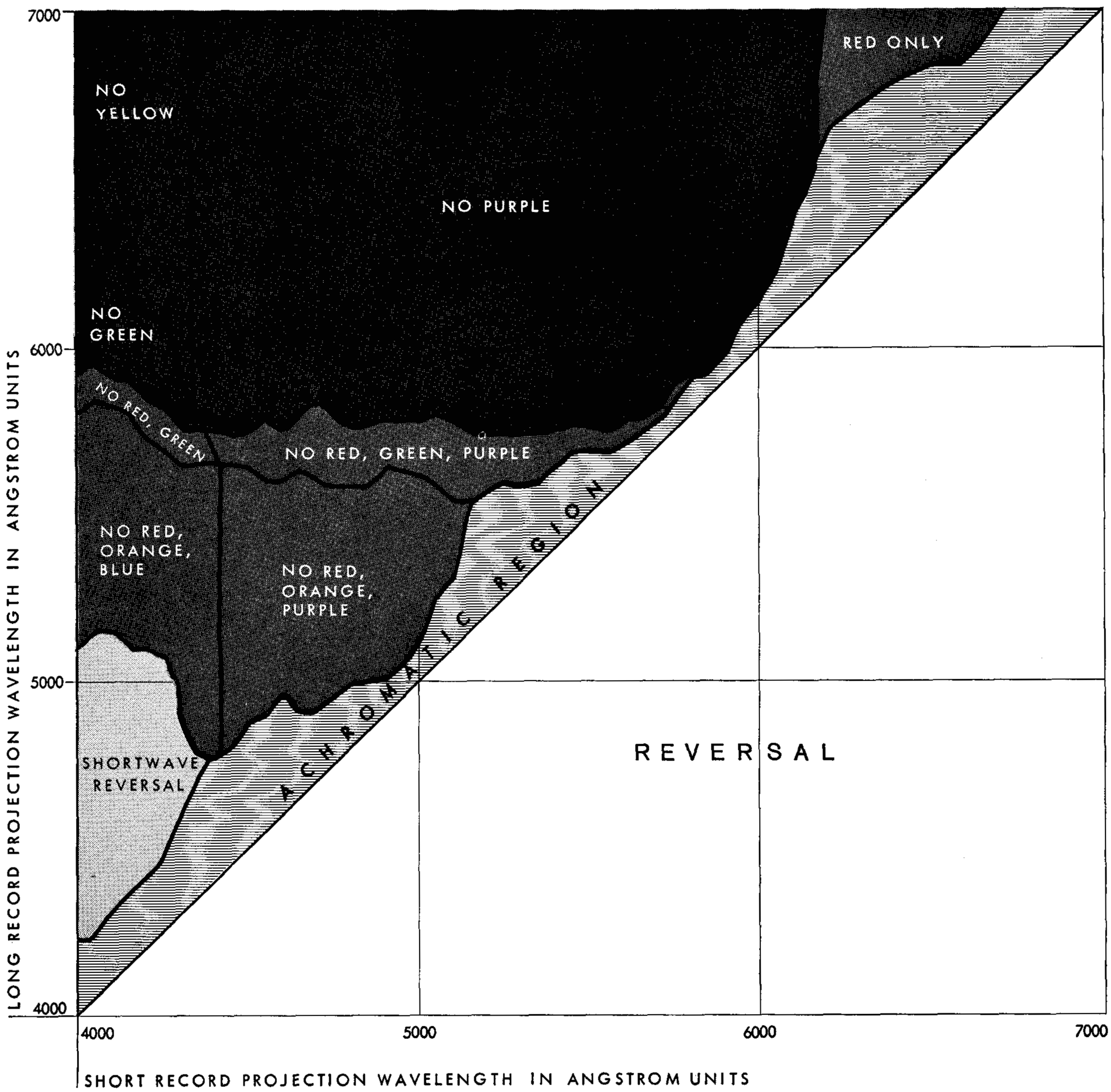


Figure 5 **Land's experimental results showing colors which are not obtained with any given pair of projection wavelengths.** *(Adapted from Ref. 2, Fig. 3.)*

3. Although it seems evident that the colors seen must be due to the relative intensities of the longer and shorter projection wavelengths, it was found that the relative intensities of the projecting beams could be varied between wide limits without unduly disturbing the colors.

4. Land found that chromatic effects were observed only when the records were of normal random-type scenes. Attempts to see colors by producing systematic variations of projection intensities, for example by means of neutral wedges, all failed—the only colors produced were those predicted by Newtonian theory.

The theory to be developed in the next sections of this paper will be found to explain all these results of Land, if not in precise numerical detail, then in a good, *qualitative* way. The classical Young-Helmholtz theory will be retained for this purpose, but it will be shown that, under the conditions of Land's experiments, a weighting factor must be applied to each of the three color receptor systems.

A mathematical analysis of the Land experiment

Let us consider a point P in the object space which is to be photographed. The different wavelengths contained in the white light which falls on the point P will be reflected and absorbed to different extents; the color of the object will depend on that portion of the spectrum which is best reflected. However, most of the naturally occurring pigments are not very selective in their reflectivity, and in Fig. 6 is shown a typical curve which relates the fraction of incident radiation which is reflected as a function, $f(\lambda)$, of the wavelength.

The object field is photographed through a filter for which the fractional transmission, $T(\lambda)$, is also a function of wavelength; a typical transmission characteristic curve is shown in Fig. 7. Thus the total energy per unit area from the point P which passes through the filter into the camera is proportional to

$$E_P=\int_\lambda f(\lambda)T(\lambda)\,d\lambda\,,$$

where the integration is carried out over the range of λ contained in the visible spectrum. Since the transmission characteristics of the filters used are quite sharp, the function $T(\lambda)$ nearly always falls off to zero in a short range of values of λ, and the limits of integration can be replaced by zero and infinity.

The transmission of the finally produced positive at the point corresponding to P will be an increasing function of E_P, the function depending on the conditions of exposure and development during the various photographic processes. Let us assume for simplicity that the transmission is proportional to E_P. (Actually this is quite feasible, and in the field of X-ray crystallography there is a photographic technique which seeks to achieve this condition.) When the positive is projected with light of wavelength L, the light falling at the point on the screen corresponding to P will have intensity proportional to E_P. If we now assume that the (white) screen has the same reflectivity for all wavelengths, then the light received by the eye from the point P on the screen will also have intensity proportional to E_P, say KE_P, and be of wavelength L. The Young-Helmholtz theory will now be invoked, and we shall assume that the stimuli per unit energy as functions of wavelength are given by $\psi_R(\lambda)$, $\psi_G(\lambda)$ and $\psi_B(\lambda)$ for the red, green and blue sensors, respectively.

The red, green and blue stimuli are thus given by

$$x_R=KE_P\psi_R(L)=K\psi_R(L)\int_{\lambda=0}^{\infty} f(\lambda)T(\lambda)\,d\lambda\,,$$

$$x_G=KE_P\psi_G(L)=K\psi_G(L)\int_{\lambda=0}^{\infty} f(\lambda)T(\lambda)\,d\lambda\,,$$

and

$$x_B=KE_P\psi_B(L)=K\psi_B(L)\int_{\lambda=0}^{\infty} f(\lambda)T(\lambda)\,d\lambda\,. \qquad (1)$$

In the Land arrangement there are two photographs taken with filters with different transmission characteristics, say $T_1(\lambda)$ and $T_2(\lambda)$, and projected with monochromatic beams of light of different wavelengths, say L_1 and L_2. The total stimulus of the red sensor will now be

$$X_R=K_1\psi_R(L_1)\int_{\lambda=0}^{\infty} f(\lambda)T_1(\lambda)\,d\lambda + K_2\psi_R(L_2)\int_{\lambda=0}^{\infty} f(\lambda)T_2(\lambda)\,d\lambda \qquad (2)$$

with corresponding expressions for X_G and X_B.

At this stage Eq. (2) will take on more meaning if we are more specific about the forms of the various functions involved. It is convenient mathematically, and *qualitatively* correct, *to assume that the functions are all Gaussian in form*, so that $f(\lambda)=\alpha_m\exp\{-\phi(\lambda_m-\lambda)^2\}$, where λ_m is the peak wavelength giving the object its color,

$$T_1(\lambda)=\beta_1\exp\{-\tau_1(\lambda_1-\lambda)^2\}\,,$$

$$T_2(\lambda)=\beta_2\exp\{-\tau_2(\lambda_2-\lambda)^2\}\,,$$

where λ_1 and λ_2 are the peak transmission wavelengths which give the filters their color,

$$\psi_R=\gamma_R\exp\{-\chi\,(\lambda_R-\lambda)^2\}\,,$$

Figure 6 **Reflectivity characteristics for a typical pigment.**

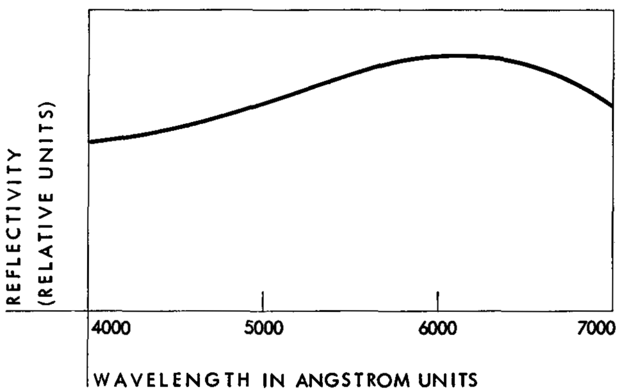
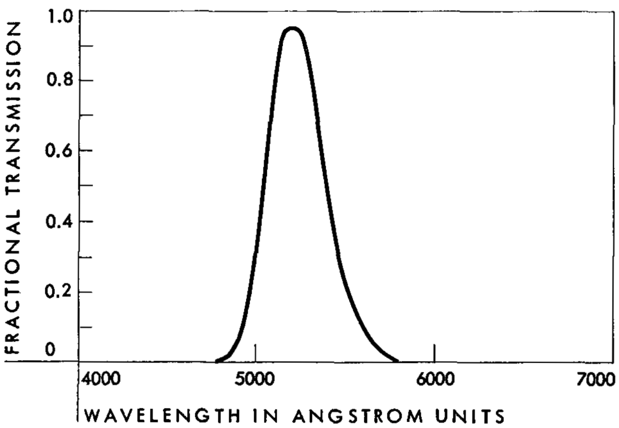


Figure 7 **Transmission characteristic for a typical filter.**



$$\psi_G = \gamma_G \exp\{-\chi_G(\lambda_G - \lambda)^2\},$$

and

$$\psi_B = \gamma_B \exp\{-\chi_B(\lambda_B - \lambda)^2\},$$

where λ_R, λ_G and λ_B are the peak-stimulus wavelengths for the three sensors.

We now find from Eq. (2)

$$X_R = K_1\alpha_m\beta_1\gamma_R \sqrt{\frac{\pi}{\phi+\tau_1}}\, F\left\{\frac{\phi\lambda_m+\tau_1\lambda_1}{(\phi+\tau_1)^{\frac{1}{2}}}\right\} \exp\left\{-\frac{\phi\tau_1(\lambda_m-\lambda_1)^2}{\phi+\tau_1}\right\} \exp\{-\chi_R(\lambda_R-L_1)^2\}$$
$$+K_2\alpha_m\beta_2\gamma_R \sqrt{\frac{\pi}{\phi+\tau_2}}\, F\left\{\frac{\phi\lambda_m+\tau_2\lambda_2}{(\phi+\tau_2)^{\frac{1}{2}}}\right\} \exp\left\{-\frac{\phi\tau_2(\lambda_m-\lambda_2)^2}{\phi+\tau_2}\right\} \exp\{-\chi_R(\lambda_R-L_2)^2\},$$

$$\text{where } F(x) = (2\pi)^{-\frac{1}{2}} \int_{t=x}^{\infty} \exp(-\tfrac{1}{2}t^2)\,dt. \qquad (3)$$

In any normal circumstances $\phi \ll \tau_1$ or τ_2, that is to say that the transmission characteristics of the filters are much sharper than the reflectivity characteristics of the pigment. Equation (3) can now be simplified to

$$X_R = B_1\alpha_m\gamma_R \exp\{-\phi(\lambda_m-\lambda_1)^2\}\exp\{-\chi_R(\lambda_R-L_1)^2\}$$
$$+B_2\alpha_m\gamma_R \exp\{-\phi(\lambda_m-\lambda_2)^2\}\exp\{-\chi_R(\lambda_R-L_2)^2\},$$

$$\text{where } B_1 = K_1\beta_1 \sqrt{\frac{\pi}{\tau_1}}\, F(\tau_1^{\frac{1}{2}}\lambda_1)$$

$$\text{and } B_2 = K_2\beta_2 \sqrt{\frac{\pi}{\tau_2}}\, F(\tau_2^{\frac{1}{2}}\lambda_2).$$

The values of B_1 and B_2 depend only on the filter characteristics and on the relative intensities of the projection beams, and we may assume for initial simplicity that the latter are adjusted so that $B_1 = B_2$. The effect of changing the relative intensities of the projection beams will be investigated later.

The relative red, green and blue stimuli are given by

$$X_R : X_G : X_B$$
$$= \gamma_R[\exp\{-\phi(\lambda_m-\lambda_1)^2\}\exp\{-\chi_R(\lambda_R-L_1)^2\}$$
$$+\exp\{-\phi(\lambda_m-\lambda_2)^2\}\exp\{-\chi_R(\lambda_R-L_2)^2\}]$$
$$:\gamma_G[\exp\{-\phi(\lambda_m-\lambda_1)^2\}\exp\{-\chi_G(\lambda_G-L_1)^2\}$$
$$+\exp\{-\phi(\lambda_m-\lambda_2)^2\}\exp\{-\chi_G(\lambda_G-L_2)^2\}]$$
$$:\gamma_B[\exp\{-\phi(\lambda_m-\lambda_1)^2\}\exp\{-\chi_B(\lambda_B-L_1)^2\}$$
$$+\exp\{-\phi(\lambda_m-\lambda_2)^2\}\exp\{-\chi_B(\lambda_B-L_2)^2\}]. \qquad (4)$$

We shall now consider an arbitrary numerical example, where

$\lambda_1 = 5{,}300$ A, a green filter,

$\lambda_2 = 6{,}500$ A, a red filter,

$L_1 = 5{,}000$ A, a blue-green projection,

$L_2 = 5{,}500$ A, a green projection,

$\lambda_R = 6{,}200$ A, the red peak,

$\lambda_G = 5{,}400$ A, the green peak,

$\lambda_B = 4{,}600$ A, the blue peak,

$\phi = 10^{-7}$, a diffusely reflecting pigment,

$\chi_R = \chi_G = \chi_B = 10^{-6}$, a figure chosen because it agrees in order of magnitude with published hypothetical stimulus curves[4] and $\gamma_R = \gamma_G = \gamma_B = 1$, which makes the areas under the curves equal.

The values of X_R, X_G and X_B are now calculated for a number of different λ_m. From the figures in Table 1, it will be seen that, whatever the object color, the green sensors are most stimulated and the red the least, and the Young-Helmholtz theory would suggest that, whatever the color of the original object, it will be seen as green to green-blue in the projection. We shall now consider a new factor which, in conjunction with the Young-Helmholtz theory, causes the eye to perceive a full range of spectral hues in the Land projections.

Table 1 **Absolute stimuli of the color sensors.**

λ_m	*Color*	X_R	X_G	X_B
4700 A	Blue	0.671	1.538	1.143
5300	Green	0.768	1.708	1.237
5850	Yellow	0.817	1.775	1.252
6100	Orange	0.826	1.773	1.236
6700	Red	0.806	1.685	1.142

Color transformation

A visual phenomenon which has been studied by many experimental psychologists is that of color transformation (also known as "color constancy").[5-11] It has been observed that although the chromatic character of the radiant energy falling on a colored object, and therefore the character of the radiation reflected by the object, may vary between wide limits, the eye tends to see the true color of the object. The observer is able in some way to correct for the chromaticity of the illumination when judging colors; it is a common experience that colors are seen quite well in candlelight, although this illumination is very yellowish. The author has found an even more striking way by which the reader may observe for himself the phenomenon of color transformation. Some green-tinted glass (a sunglass lens will do) is held at arm's length and a colored object, preferably a yellow or a blue, is seen through it. The object color will be judged to be green if the green tint is strong enough. When the same object is now viewed with the glass next to the eye, so that the whole field of view is green-tinted, the object color will be seen and judged to be its normal color. The actual character of the light proceeding from the object to the eye would be the same in each case; however, the first judgment of color was made in a generally white-illuminated field of view while the second was made in relation to a green-tinted (equivalent to green-illuminated) field of view. The color transformation effect can also be seen with a white object; with the tinted glass held close to the eye the object is seen to be white but with a slight tinge of the color of the glass.

In a series of experiments carried out by the author it was found that color transformation did not occur when objects were illuminated with monochromatic light. To anyone familiar with a sodium-lamp illuminated street scene, this will be no surprise. All colors appear as shades of grey, from white to black, with a strong yellow coloration superimposed. However, when the objects are illuminated simultaneously by two different monochromatic light sources, color transformation will be observed. Not all colors can normally be seen, and those which are seen are mostly pale and unsaturated, a situation which seems similar to that occurring with the Land projections. It will be shown that, from the eye's standpoint, the situations are indeed analogous and hence that color transformation should be expected for the pictures seen in the Land experiments. First, however, one of Land's results may be explained in terms of the work carried out by the experimental psychologists. They showed that color transformation could only be observed when the field of view was sufficiently detailed — such as is the case with a normal random-type scene.[12] Land's experiments with neutral wedges produced a field of view with variations in the relative intensities of the projection beams, but with no detail, and, as expected, no color transformation was observed.

We now consider an object, with reflectivity curve as shown by Fig. 6, illuminated by monochromatic light of wavelength L_1. The object will reflect only light of wavelength L_1 with an intensity proportional to $f(L_1)$. The resultant stimuli of the red, green and blue sensors will be

$$x_R = K_1 f(L_1)\psi_R(L_1),$$

$$x_G = K_1 f(L_1)\psi_G(L_1),$$

and

$$x_B = K_1 f(L_1)\psi_B(L_1),$$

where K_1 is a constant of proportionality. The relative values of these stimuli are $\psi_R(L_1):\psi_G(L_1):\psi_B(L_1)$, which depends on the illuminating wavelength but not on the color of the illuminated object. The only effect of $f(L_1)$ is to make the absolute values of the stimuli greater or less, that is, to make the object appear brighter or less bright, and hence all colors will be judged as shades of grey overlaid with the illuminating color.

When the scene is illuminated by light of two different wavelengths, L_1 and L_2, the total red, green and blue stimuli are

$$X_R = K_1 f(L_1)\psi_R(L_1) + K_2 f(L_2)\psi_R(L_2),$$

$$X_G = K_1 f(L_1)\psi_G(L_1) + K_2 f(L_2)\psi_G(L_2),$$

and

$$X_B = K_1 f(L_1)\psi_B(L_1) + K_2 f(L_2)\psi_B(L_2). \tag{5}$$

When the proportions of X_R, X_G and X_B are taken in this case, the terms $f(L_1)$ and $f(L_2)$ are now involved and the proportions will depend on the color of the object. The eye is now receiving stimuli capable of yielding color information if only it possesses the necessary interpretive mechanism and the experimental fact of color transformation shows that this mechanism actually exists. A comparison of Eqs. (2) and (5) shows that the effect on the eye of the Land projections is similar to that of the illuminated objects, and the inference from this is that the eye carries out a color transformation on the projected pictures.

To determine a plausible mathematical model of color transformation, we shall use the information that a white object is seen as nearly white under chromatic illumination. The reflectivity curve for a white object is given by the equation $f(\lambda) =$ a constant. The relative stimuli for a white object are thus

$$\begin{aligned} {}_WX_R : {}_WX_G : {}_WX_B &= K_1 f(L_1)\psi_R(L_1) + K_2 f(L_2)\psi_R(L_2) \\ &: K_1 f(L_1)\psi_G(L_1) + K_2 f(L_2)\psi_G(L_2) \\ &: K_1 f(L_1)\psi_B(L_1) + K_2 f(L_2)\psi_B(L_2). \end{aligned} \tag{6}$$

However, it is known that under normal viewing conditions, with white light, the three stimuli should be equal. This suggests that for the white object to be seen as white the effective stimulus for each sensor is found by dividing the actual stimulus by itself. The first proposed mechanism for converting any stimulus to its *effective* value after color transformation is to divide it by the corresponding stimulus for a white object. Thus corresponding to (4), the relative values of the *effective* stimuli are given by

$$\begin{aligned} Y_R : Y_G : Y_B &= \frac{X_R}{{}_WX_R} : \frac{X_G}{{}_WX_G} : \frac{X_B}{{}_WX_B} \\ &= \frac{\exp\{-\phi(\lambda_m-\lambda_1)^2\}\exp\{-\chi_R(\lambda_R-L_1)^2\}+\exp\{-\phi(\lambda_m-\lambda_2)^2\}\exp\{-\chi_R(\lambda_R-L_2)^2\}}{\exp\{-\chi_R(\lambda_R-L_1)^2\}+\exp\{\chi_R(\lambda_R-L_2)^2\}} \\ &: \frac{\exp\{-\phi(\lambda_m-\lambda_1)^2\}\exp\{-\chi_G(\lambda_G-L_1)^2\}+\exp\{-\phi(\lambda_m-\lambda_2)^2\}\exp\{-\chi_G(\lambda_G-L_2)^2\}}{\exp\{-\chi_G(\lambda_G-L_1)^2\}+\exp\{\chi_G(\lambda_G-L_2)^2\}} \\ &: \frac{\exp\{-\phi(\lambda_m-\lambda_1)^2\}\exp\{-\chi_B(\lambda_B-L_1)^2\}+\exp\{-\phi(\lambda_m-\lambda_2)^2\}\exp\{-\chi_B(\lambda_B-L_2)^2\}}{\exp\{-\chi_B(\lambda_B-L_1)^2\}+\exp\{\chi_B(\lambda_B-L_2)^2\}} \end{aligned} \tag{7}$$

Table 2 **Transformed stimuli with white object seen as white.**

λ_m	*Color*	Y_R	Y_G	Y_B
4700 A	Blue	0.792	0.839	0.882
5300	Green	0.908	0.933	0.954
5850	Yellow	0.966	0.966	0.965
6100	Orange	0.976	0.964	0.953
6700	Red	0.953	0.915	0.881

for the case we have previously considered, and these are the stimuli which will be used from the point of view of the Young-Helmholtz theory.

If the values of the absolute stimuli in Table 1 are now transformed to Y's, the effective stimuli, we obtain the data given in Table 2. An examination of these figures reveals that the red object would certainly be seen as red with the transformed stimuli whereas with the absolute stimuli, the X's, it would be seen as green. To find the hue and saturation seen for each color we recall the rule given in the section on the Young-Helmholtz theory. The value of λ is found from the equations

$$Y_R - Z = A\gamma_R \exp\{-\chi_R(\lambda_R-\lambda)^2\},$$

$$Y_G - Z = A\gamma_G \exp\{-\chi_G(\lambda_G-\lambda)^2\},$$

and

$$Y_B - Z = A\gamma_B \exp\{-\chi_B(\lambda_B-\lambda)^2\}, \tag{8}$$

that is to say, when a constant background Z is subtracted from each stimulus, the remainders are proportional to the stimuli for some wavelength λ.

From Eqs. (8) we find

$$\frac{Y_B-Y_G}{Y_B-Y_R} = \frac{\gamma_B \exp\{-\chi_B(\lambda_B-\lambda)^2\} - \gamma_G \exp\{-\chi_G(\lambda_G-\lambda)^2\}}{\gamma_B \exp\{-\chi_B(\lambda_B-\lambda)^2\} - \gamma_R \exp\{-\chi_R(\lambda_R-\lambda)^2\}} = \xi. \tag{9}$$

To find λ for a given value of ξ a graph is constructed (Fig. 8) showing ξ as a function of λ for the numerical example under consideration.

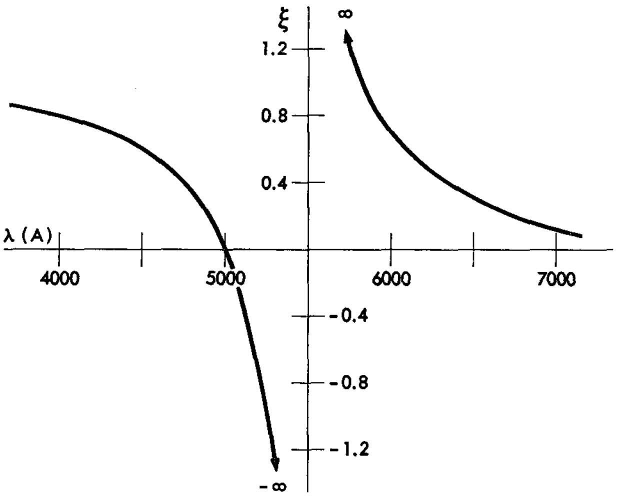


Figure 8 $\xi\left(=\dfrac{Y_B-Y_G}{Y_B-Y_R}\right)$ **as a function of** λ.

Thus for the red object

$$\frac{Y_B-Y_G}{Y_B-Y_R} = \frac{0.881-0.915}{0.881-0.953} = 0.473$$

and from the chart the possible values of λ are 4650 A and 6220 A. To decide which of these is appropriate, we note that if $Y_R \lessgtr Y_B$, then $\lambda \lessgtr \frac{1}{2}(\lambda_R+\lambda_B)$, so that in this case the hue will be that of the spectral color 6220 A, a fairly good red.

It will be found, however, that the ratios for the other values of λ (excluding that for yellow) are 0.477, 0.456 and 0.478, almost equal to that for the red object. The value for yellow is uncertain because of the closeness of the values of Y and the limited accuracy of the calculations. It can be shown from (7) and (9) that the value of ξ is indeed actually independent of λ_m, so that the mode of color transformation suggested can lead to only one of two colors in the projection, for the numerical example we have considered either 4650 or 6220 A. This does not conform with observations of color transformation made with pairs of monochromatic illuminating light beams.

Table 3 **Transformed stimuli; white object with color tint.**

$x=$	0.005			0.01			0.05		
λ_m	Y_R	Y_G	Y_B	Y_R	Y_G	Y_B	Y_R	Y_G	Y_B
4700	0.794	0.847	0.889	0.796	0.855	0.896	0.825	0.924	0.941
5300	0.910	0.941	0.961	0.911	0.949	0.966	0.944	1.022	1.017
5850	0.966	0.975	0.973	0.969	0.985	0.979	1.003	1.065	1.028
6100	0.978	0.975	0.961	0.980	0.985	0.966	1.015	1.064	1.016
6700	0.954	0.927	0.888	0.956	0.936	0.893	0.991	1.007	0.940

The color transformation process which has just been found to be unsatisfactory is one which makes a white object appear white. We shall now consider another which makes a white object appear to have a slight tinge of the illuminating colors. This is of the form

$$Y_R = \frac{X_R}{{}_W X_R}(1 + x_W X_R), \tag{10}$$

where x is a small constant, the same for each of the color sensors. For a white object this gives

$${}_W Y_R = 1 + x_W X_R ,$$

$${}_W Y_G = 1 + x_W X_G ,$$

and

$${}_W Y_B = 1 + x_W X_B ,$$

which is equivalent to a white background of unit magnitude plus extra stimuli $x_W X_R$, $x_W X_G$ and $x_W X_B$ which would be given by equal quantities of radiation of wavelengths L_1 and L_2.

For our numerical example we carry out the new transformation for some selected values of x. (See Table 3.) These figures may be translated into the equivalent hue by calculating the values of ξ and using Fig. 8 as before. These results are shown in Table 4.

It will be seen that the projection color now depends on λ_m, which is as desired, but it also depends on the arbitrary value of x.

The value for the saturation, as defined in the section on the Y-H theory, will be $1-(3Z/Y_R+Y_G+Y_B)$, where the value for Z may be found from Eqs. (8) as

$$Z = \frac{Y_G \gamma_R \exp\{-\chi_R(\lambda_R-\lambda)^2\} - Y_R \gamma_G \exp\{-\chi_G(\lambda_G-\lambda)^2\}}{\gamma_R \exp\{-\chi_R(\lambda_R-\lambda)^2\} - \gamma_G \exp\{-\chi_G(\lambda_G-\lambda)^2\}} .$$

The value of Z for the red object, with $x=0.01$, is 0.843 and the saturation of the color seen is thus $1-2.529/2.785 = 0.093$ or 9.3%. This represents a very low saturation, in agreement with Land's observations. However, the actual figure is probably not very significant, depending as it does on the assumed forms of the functions $f(\lambda)$, $T(\lambda)$ and $\psi(\lambda)$ and the value of x.

Another case which may occur is when Y_G is the least of the three stimuli. This corresponds to a nonspectral purple hue; a value of Z is now found, greater than any of the stimuli, so that $Z-Y_R$, $Z-Y_G$ and $Z-Y_B$ are proportional to the stimuli for some spectral color. The purple will be the complement of this spectral color. The saturation for this case can be defined as $1-(Y_R+Y_G+Y_B/3Z)$. The complementary color λ_C satisfies the equations

$$Z - Y_R = A\gamma_R \exp\{-\chi_R(\lambda_R-\lambda_C)^2\} ,$$

$$Z - Y_G = A\gamma_G \exp\{-\chi_G(\lambda_G-\lambda_C)^2\} ,$$

$$Z - Y_B = A\gamma_B \exp\{-\chi_B(\lambda_B-\lambda_C)^2\} ,$$

which leads to the same Eq. (9) but with λ_C in place of λ. Thus Fig. 8 may be used to solve for λ_C in terms of the Y's, except that in this case it should be noted that if

$$Y_R \lesseqgtr Y_B, \text{ then } \lambda_C \gtreqless \tfrac{1}{2}(\lambda_R+\lambda_B) .$$

If in our numerical example a purple object is assumed to have a reflectivity curve which is the sum of those for the blue and red objects, then the effective stimuli will be the sum of the effective stimuli for the blue and red objects. Thus for a purple object, the figures in Table 5 apply. It will be noticed that purple is seen as blue or green in this case.

Table 4 **Hue seen with variation in white tint.**

	Hue, in angstrom units		
$\lambda_m/x=$	0.005	0.01	0.05
4700	4690	4720	4930
5300	4740	4810	5190
5850	5090	5160	5330
6100	5880	5660	5400
6700	6070	5990	5690

A comparison of theory and experiment

The IBM 704 computer has been used to calculate the hue and saturation for various values of λ_m over ranges of projection wavelengths covering the whole visible spectrum. The various constants used in the calculation were

$\lambda_1 = 5300$ A, $\lambda_2 = 6500$ A — Green and red filters.

$\phi = 10^{-7}$ — A diffusely reflecting object.

Table 5 **Hue seen for purple object with variation in white tint.**

$x=$	0.005			0.01			0.05		
	Y_R	Y_G	Y_B	Y_R	Y_G	Y_B	Y_R	Y_G	Y_B
4700-6700 A	1.748	1.774	1.777	1.752	1.791	1.789	1.816	1.931	1.881
Hue	4950 A			4960 A			5200 A		

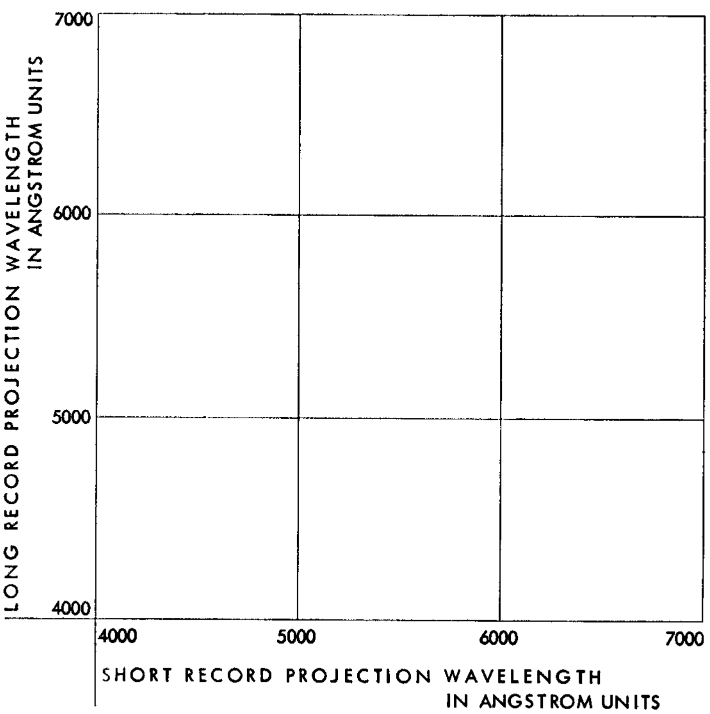
$\lambda_B = 4700$ A
$\lambda_G = 5500$ A
$\lambda_R = 5900$ A
$\chi_B = 0.0000081$
$\chi_G = 0.0000049$
$\chi_R = 0.0000016$
$\gamma_B = 9.0$
$\gamma_G = 7.0$
$\gamma_R = 4.0$
$x = 0.01$

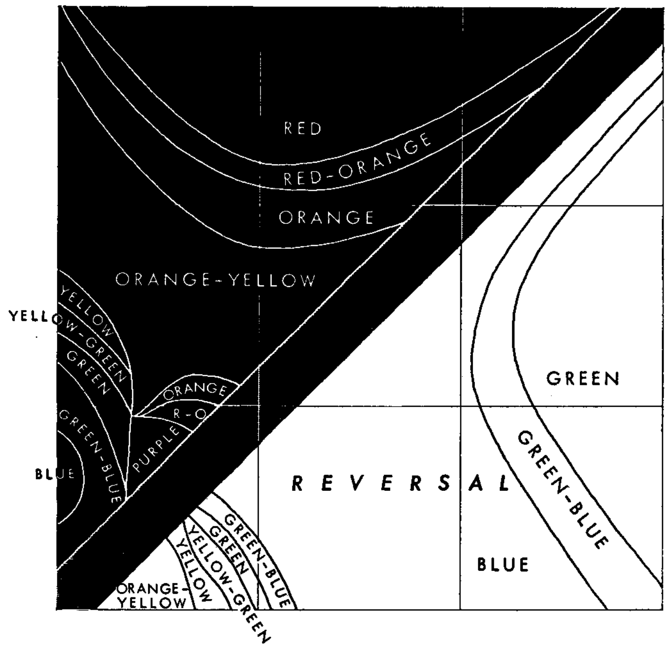
These figures give reasonable qualitative agreement with the curves in Fig. 2.

The results of the hue calculations are shown graphically in Fig. 9. The color limits are taken as

red	over 6200 A
reddish orange	6050 - 6200 A
orange	5950 - 6050 A
orange-yellow	5850 - 5950 A
yellow	5750 - 5850 A
yellow-green	5500 - 5750 A
green	5200 - 5500 A
green-blue	4900 - 5200 A
blue	below 4900 A .

Below: Ordinate and abscissa values for the Charts of Figure 9

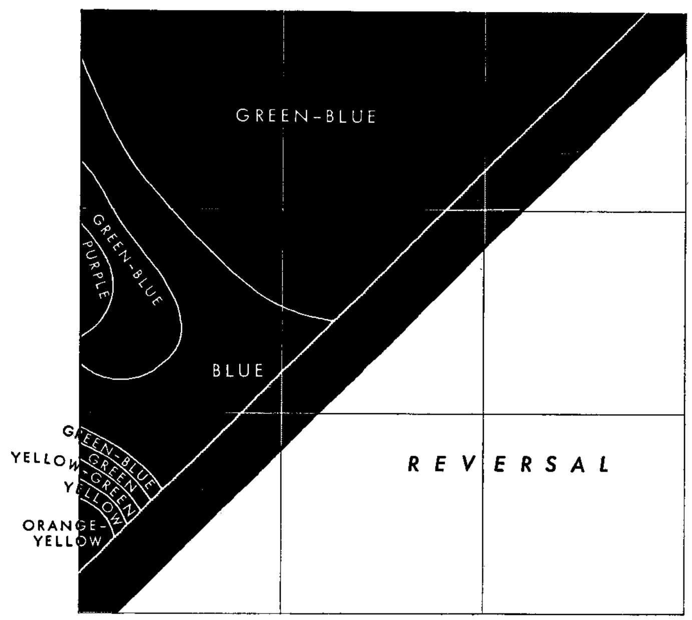


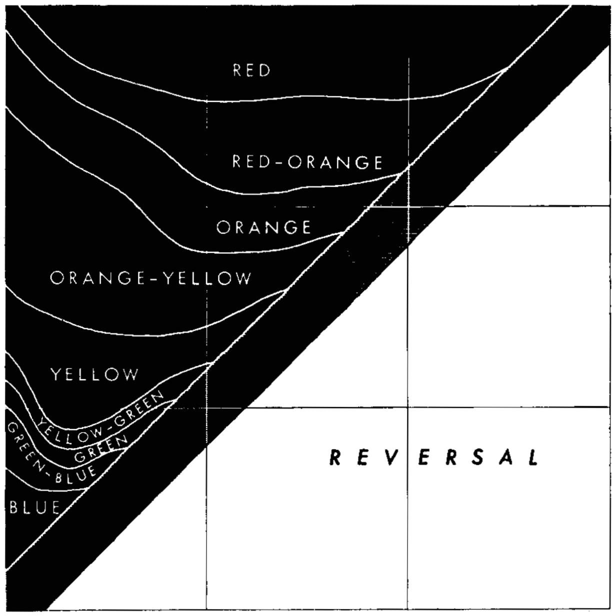


Red ($\lambda = 6700$ *angstrom units*)

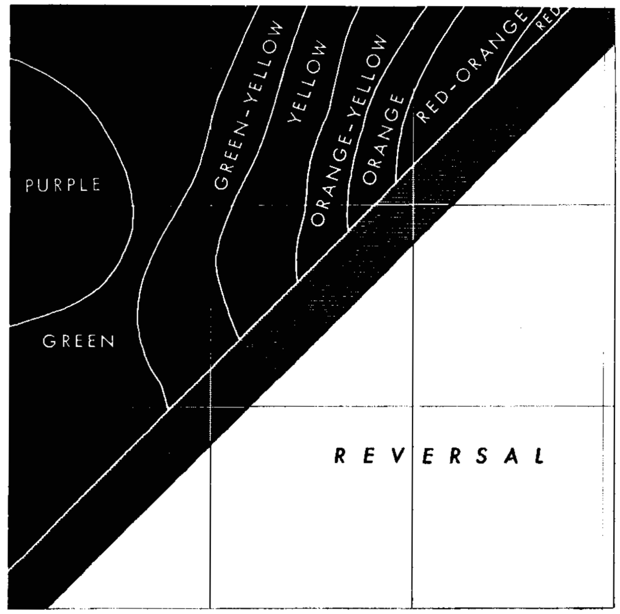
Figure 9 **Colors seen in projection with the proposed theoretical transformation shown for variously colored objects as functions of the projection wavelengths.**

Green ($\lambda = 5300$ *angstrom units*)



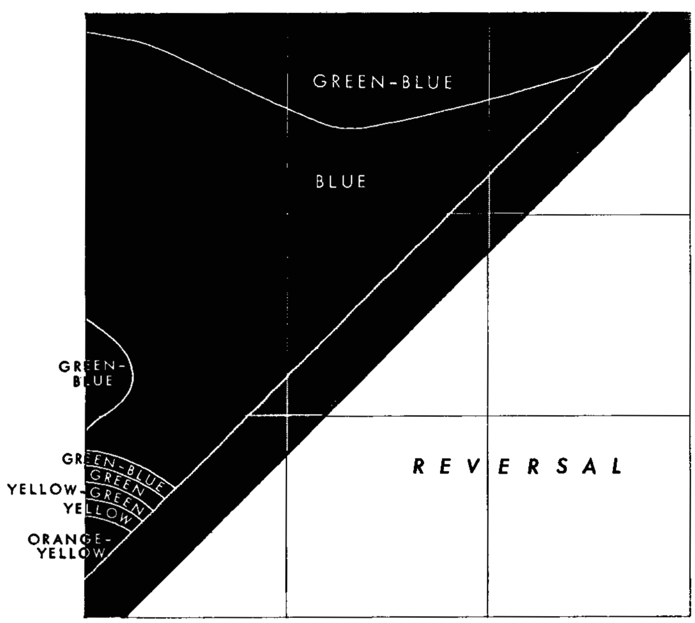


Orange (λ = *6000 angstrom units*)



Yellow (λ = *5800 angstrom units*)

Blue (λ = *4700 angstrom units*)



Purple (λ = *4700* + *6700 angstrom units*)

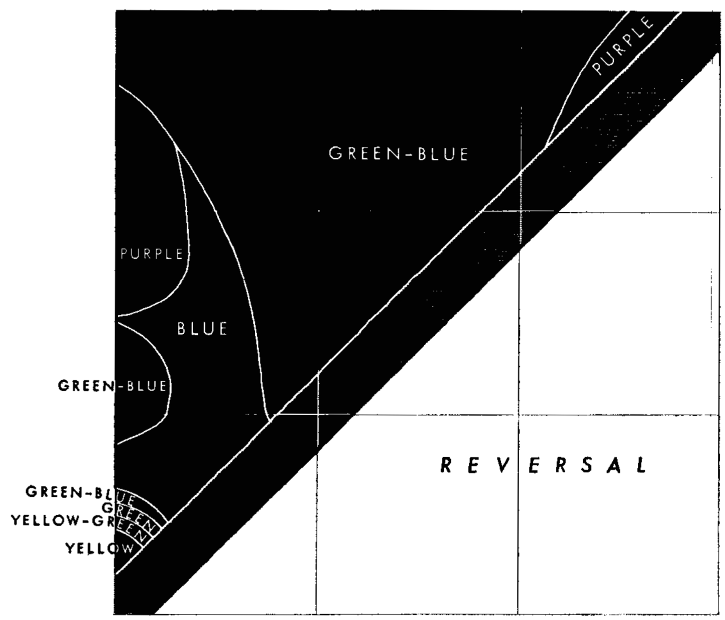


Table 6 **Range of intensities for color stability.**

λ_m	*Color*	*Approximate range over which seen*	*Range given by Land*
4700	Blue	8:1	10:1
5300	Green	4:1	6:1
5850	Yellow	5:1	30:1
6100	Orange	3:1	5:1
6700	Red	2:1	5:1

If we compare the results for the red object with Fig. 5 (Land's data[3]), it will be seen that there is rough qualitative agreement as to the areas where red may be seen. In addition, it will be noted that the calculated results reproduce the area of color reversal which occurs when both projection wavelengths are short. This occurs as a result of the comparative sharpness of the blue stimulus (Fig. 2). For two wavelengths, each less than about 4600 A, it is possible that the relative proportions of red and green stimuli, compared with the blue, can be greater for the shorter wavelength. This leads to color reversal.

The general agreement between the calculated and experimental results is also found for the colors corresponding to other values of λ_m, including purple.

The saturation diagrams all show along the diagonal, where the two projection wavelengths are equal or nearly so, a region of very low saturation which corresponds to the achromatic region found by Land. An interesting feature which occurs in the experimental diagram is the presence of the three bulges in the achromatic region. One is tempted to relate the position of these to the three peaks of the stimulus curves. However, they do not appear on the calculated saturation charts; the chart for a green object is shown in Fig. 10.

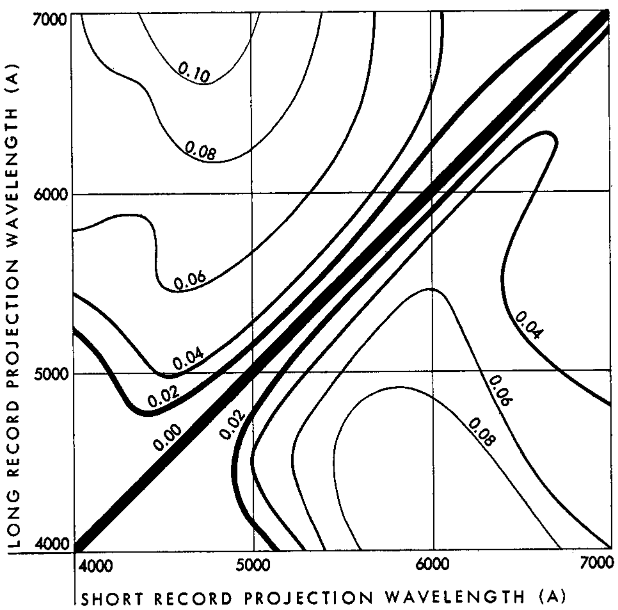
The effect of changing the relative intensities of the projection beams has also been investigated. Relation (7) is rewritten in the general form

Variations of N_1 and N_2 correspond to variations of the projection intensities. The hue is calculated for various λ_m, with the projection wavelengths 4500 A and 5750 A (as used by Land in his experimental work) and with N_1 and N_2 taking all pairs of values from 1, 2, 4, 8, 16, 32, 64, 128. The general rules for color stability which have been interpolated, compared with those found by Land, are given in Table 6.

White as a projection color

Normal white light contains all wavelengths within the visible range and, as a rough approximation, we may assume that the intensity-wavelength relationship for white light is as shown in Fig. 11. If the two chosen projection beams are white and, let us say, red, then most of the white light spectral components will be of shorter wavelength than the red. If the white light is then used to project the short record, a normally colored image should result. If, on the other hand, the monochromatic projection beam was a blue, then it would follow that the

Figure 10 **The saturation diagram for a green object.**



$$Y_R:Y_G:Y_B=\frac{N_1\exp\{-\phi(\lambda_m-\lambda_1)^2\}\exp\{-\chi_R(\lambda_R-L_1)^2\}+N_2\exp\{-\phi(\lambda_m-\lambda_2)^2\}\exp\{-\chi_R(\lambda_R-L_2)^2\}}{N_1\exp\{-\chi_R(\lambda_R-L_1)^2\}+N_2\exp\{-\chi_R(\lambda_R-L_2)^2\}},$$

$$:\frac{N_1\exp\{-\phi(\lambda_m-\lambda_1)^2\}\exp\{-\chi_G(\lambda_G-L_1)^2\}+N_2\exp\{-\phi(\lambda_m-\lambda_2)^2\}\exp\{-\chi_G(\lambda_G-L_2)^2\}}{N_1\exp\{-\chi_G(\lambda_G-L_1)^2\}+N_2\exp\{-\chi_G(\lambda_G-L_2)^2\}},$$

$$:\frac{N_1\exp\{-\phi(\lambda_m-\lambda_1)^2\}\exp\{-\chi_B(\lambda_B-L_1)^2\}+N_2\exp\{-\phi(\lambda_m-\lambda_2)^2\}\exp\{-\chi_B(\lambda_B-L_2)^2\}}{N_1\exp\{-\chi_B(\lambda_B-L_1)^2\}+N_2\exp\{-\chi_B(\lambda_B-L_2)^2\}}.$$

white light should be used to project the long record to obtain a normally colored picture.

For a second monochromatic beam of general wavelength λ the white light would be used for the long or short record projection, depending on whether λ is more towards the short end or the long end of the spectrum. Let us say that $\lambda = 5000$ A and that the white light is used to project the long record. Referring to Fig. 11, all the components of the white light greater than 5000 A in wavelength will contribute to give a normally colored picture while the remainder will tend to give color reversal; the net result will be a normally colored image. A monochromatic light of wavelength about 5800 A divides the spectrum into two equivalent portions such that no net chromatic effects can occur with white light projecting either the long or the short record.

Land found that it was possible to photograph red and white projections and to obtain pictures giving similar color effects to those seen by the eye. It is clear that something tantamount to a color transformation must be taking place with the color film. A blue object will be projected primarily with the white light and with less of the red, a red object mostly with the red light and with less of the white. It is fairly simple with some types of color film to use it in such a way that it becomes blue-sensitive. In this case, white will photograph as blue, red as red and intermediate pink tints will photograph as intermediate spectral hues. The greater the number of photographic or other reproduction processes carried out, the stronger can this effect become, and Land's ability to photograph the red and white projections in full color is quite understandable.

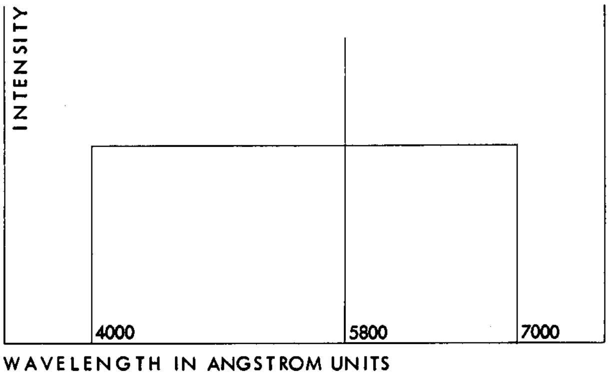


Figure 11 **The intensity-wavelength distribution for a white light source.**
With a white projection beam for the long record, a monochromatic beam of 5000 A, for example, would serve as the short record.

Conclusions

The agreement between the experimental and theoretical results calculated on the IBM 704 is satisfactory enough for the suggested mechanism of color transformation to be regarded as at least plausible. The Land results are therefore seen to be consonant with the Young-Helmholtz theory inasmuch as the Y-H theory applies to the perception of color in white-light illumination. For a more general theory of color perception, with any generally chromatic illumination, the additional factor of color transformation must be recognized.

The actual mechanism of color transformation and the general agreement of calculated and experimental results is perhaps less important than the demonstration that the Land experimental arrangement affects the eye in a similar way to another arrangement, in which objects are seen under two monochromatic illuminants, where color transformation is observed to take place.

Certain discrepancies between the calculated and experimental results, however, suggest that the process of color transformation carried out by the eye is somewhat different from that proposed in this paper. The main features of the proposed mechanism, however, seem correct because it accounts for the perception of the full chromatic range.

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