

**The Algebraic-Topological Basis for Network Analogies and the Vector Calculus**, F. H. Branin, Jr., *Proceedings of the Symposium on Generalized Networks*, Microwave Research Institute Symposia Series 16, 453-491 (April 1966). Network analogies exist for a variety of physical systems including many described by the vector calculus. An explanation of this fact is given in terms of the algebraic-topological principles upon which both network theory and the vector calculus are based. The usual network problem is shown to consist of both a *topological* structure, called a linear graph, and an *algebraic* structure whose properties, once identified, lead directly to firm ground rules for setting up network analogies. Electrical, mechanical, and structural networks are discussed as examples. By extending the algebraic-topological treatment of linear graphs to topological structures containing surface and volume elements, a direct linkup between network theory and the vector calculus becomes possible. Two significant results of this development are: (1) a novel topological interpretation of Maxwell's equations for the electromagnetic field, and (2) the validation of network representations for two large classes of partial differential equations.

## Abstracts

from recent  
papers by  
IBM authors

**An Algorithm for Generating Permutations**, G. G. Langdon, Jr., *Communications of the ACM* 10, No. 5, 298-299 (May 1967). An algorithm is described which under repeated application generates all permutations of  $K$  elements. Only the previously generated permutation, the constant  $K$ , and a temporary index are needed. Starting with a particular ordering of  $K$  elements (a b c d), repeated application of the algorithm will generate  $K-1$  additional permutations by  $K-1$  successive rotations. From the initial circular ordering of  $K$  objects, another circular ordering can be obtained by rotating the  $K-1$  lowest elements. For each new  $K-1$  circular ordering, another  $K-2$  can be obtained by rotating the  $K-2$  lowest elements. By continuing in this manner, applications of the algorithm will generate all  $(K-1)!$  circular orderings, or since each circular ordering yields  $K$  permutations, the algorithm generates all  $K!$  permutations.

**Approximations and Bounds for Eigenvalues of Elliptic Operators**,\* L. Fox,\*\* P. Henrici,\*\*\* and C. Moler,\*\*\*\* *SIAM Journal Numerical Analysis* 4, No. 1, 89-102 (1967). A method is presented to obtain rigorous bounds for the eigenvalues of certain self-adjoint elliptic partial differential operators. The main tools are the Krylov-Bogoliubov inequality, the maximum principle, and the Bergman-Vekua theory of complete systems of solutions. The method is applied to estimate the lowest ten fundamental frequencies of an L-shaped membrane. The results are far superior to those obtained by other means.

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**Automatic Information System**, R. Dubon, *L'Onde Electrique* 47, No. 482, 663-668 (May 1967). The IBM European Technical Information Center offers a centralized and computerized in-house information Retrieval and Dissemination System for IBM's scientific and technological personnel in Europe. The system uses an IBM 1401 for input/output and an IBM 7090 for searching approximately 120,000 words/minute. The answers are printouts of the text which satisfy the question logic. The technique used at La Gaude for retrospective searching and for a Current Information Selective System is a Normal Text Information Retrieval System. With each abstract the individual user receives a Port-a-Punch response card. The user then returns the response card to his local European Technical library for processing.

**Automation in Libraries: A Projection**, M. Griffin, *Canadian Library* 23, No. 5, 360-367 (March 1967). Automation is an evolutionary development which answers man's need for assistance in his manual tasks. We might say that in libraries, the first state in automation came with the printing press; the second major stage seems to be coming with the computer. As the population grows, so do the demands for information; as libraries grow, in response, so do the backlogs in processing, and the traditional methods of library organization are severely strained. The benefits of automation are needed to cope with accelerated publishing and to provide library users with better service. For the libraries, this means (in this age of technology) recognition of the fact that trial and error are fundamental to development. These precipitate needed change: the challenge is to accommodate ourselves to technological changes, adapting new techniques to speed the flow of processing of materials and information.

**Communication-System Blackout During Reentry of Large Vehicles**, F. H. Mitchell, Jr., *Proceedings of the IEEE* 55, No. 5, 619-626 (May 1967). Much of the theoretical research on reentry blackout is in a format difficult for the communications design engineer to use in his system analysis. This paper derives simplified equations for the average sheath power loss that may be added (in db) to the usual space loss to obtain an approximate total propagation loss. From these calculations, the duration of the communications blackout may be found. The plasma and sheath properties are discussed in detail, but largely without supporting mathematics, in order to give the design engineer a better understanding of the overall problem. For the same reason and to provide insight into the final results, the average radiated power is found, using both intuitive and rigorous techniques. Several graphs of plasma properties are included in the development as an aid to numerical computations, and results are compared with the work of other authors.

**Computer-Assisted Circuit Engineering**, J. A. Zumbado and J. B. Eggerling, *NASA Electronics Research Center Computer-Aided Circuit Design Seminar*, pp. 65-74 (April 11-12, 1967). This paper describes the capabilities an engineering organization must consider to efficiently perform circuit design and analysis with the aid of computers. The availability of a large computer system is assumed. Circuit engineering considers the worst-case circuit performance, which is a function of piece-part tolerances, temperature environment, and aging. Circuit performance criteria include dc stability, power dissipation, and dynamic performance. Dynamic performance encompasses transient behavior, dynamic stability, and four-terminal network characterization of module circuitry. While emphasis is given to circuit analysis and design, system design and analysis can be, and has been, performed using the basic approaches described herein. Accurate and timely circuit engineering is based upon: (a) accurate circuit parameter characterization, (b) flexible network analysis computer programs, (c) continuous computer program development, and (d) empirical investigations in adequate laboratories.

**Computer Partitioning Improves Long-Term Reliability in Space**, M. Ball and F. H. Hardie, *Space Aeronautics* 47, No. 5, 114-118 (May 1967). The ground rules and trade-offs used to electrically partition an extended-mission space computer are discussed. The configured computer used triple modular redundancy and electrical partitioning to meet computer reliability goals.

**Data Collection System Controls Manufacturing**, J. E. Kunze, *Tool and Manufacturing Engineer* 58, No. 4, 134-137 (April 1967). A computerized system based on IBM 357 off-line terminals permits rigid control of manufacturing operations, yet allows the introduction of both part handling and scheduling changes to help IBM Rochester meet its production commitments despite complex product lines, continuous engineering changes, and tight delivery schedules. This technique is called real-time dispatching. In use in the plant's machining area, the system provides dynamic control of parts manufacturing and sub-assembly operations.

**Digital Analysis Of Bursts in the Time Domain**, W. W. Lang, G. C. Maling, Jr., W. A. Taylor, Jr.,\* *IEEE Transactions on Audio and Electroacoustics* AU-15, No. 1, 15-19 (March 1967). A technique is described for analyzing bursts and burst-like events with a digital recording system and a general-purpose computer. The waveform of the burst is sampled by an analog-to-digital converter and stored on magnetic tape. The digitized samples are used as input data to an IBM 7094 data processing system for which a burst-analysis program has been written in the FORTRAN language. The analysis of amplitude and shape variations of a burst train is completed without manual processing of the data. This technique is used to analyze the acoustical noise produced by a card punch.

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**An Experimental Word Decode and Drive System for a Magnetic Film Memory with 20-ns READ-Cycle Time**, D. Seitzer, *IEEE Transactions on Electronic Computers* EC-16, No. 2, 172-179 (April 1967). Design principles of a word selection and drive scheme for a 20-ns nondestructive-READ-cycle-time flat-film memory with 150,000-bits storage capacity, are presented and verified by cross-sectional tests. The word lines are matched at one end and driven from the other by a high-speed driver building block compatible with integrated circuit technology which is connected to a decode matrix. Suitable decode matrix line drivers for providing pulses of different width and amplitude for READ operations up to 50 MHz and WRITE operations up to 20 MHz, respectively, are described. Experiments with a cross-sectional setup for 64 outputs prove the feasibility of the system. Its operation in connection with a memory model indicates an access time of 30 ns.

**Frequency Demultiplexing Using Computer Techniques**, A. Piscopo, *Telemetry Journal* 2, No. 3, 42-48 (April-May 1967). The demultiplexing of an FM/FM signal is demonstrated through the use of correlation techniques. Equivalent computer functions are derived for both the bandpass filter and FM discriminator used in an Inter-Range Instrumentation Group (IRIG) telemetry ground station. Emphasis is on functional concepts and the use of a general-purpose computer, IBM SYSTEM/360. Results are presented in the form of computer plots. Full implementation of these concepts would require further study in both the engineering and programming areas.

**A Graphic-Output Adapter for Remote Plotting**, E. T. Johnson, *Eighth National Symposium of the Society for Information Display*, (May 24, 1967). An experimental graphic-output adapter has been developed for use with the IBM 1051 and 2741 typewriter terminals which receive and send information over voice-grade telephone lines. The adapter can drive a digital plotter, an analog plotter, or a storage CRT, asynchronously. It can draw both points and X, Y, 45° vectors; furthermore, it offers a flexible, potentially low-cost alternative to on-line display systems.

**How Long is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension**, B. Mandelbrot, *Science* 156, No. 3775, 636-638 (May 5, 1967). Geographical curves are so involved in their detail that their lengths are often infinite or, rather, undefinable. However, many are statistically "self-similar," meaning that each portion can be considered a reduced-scale image of the whole. In that case, the degree of complication can be described by a quantity  $D$  that has many properties of a "dimension," though it is fractional; that is, it exceeds the value unity associated with the ordinary, rectifiable, curves.

**Measurement and Simulation of the Computer Environment**, T. W. Steading and J. N. Paulson, *Institute of Environmental Sciences National Technical Meeting*, pp. 275-283 (April 10, 1967). This paper describes three field investigations made by IBM which resulted in better definitions of earth environmental parameters of man-operated commercial and industrial computers. The environments of interest were floor vibration, airborne dust, and conducted electrical switching transient noise. The techniques used in defining these environments, as well as experimental methods used in simulating them, are described. General summaries derived from the collected data are presented.

**Modular Approach to System Design**, Walter J. Schuelke, *Automation* 14, No. 4, 77-83 (April 1967). This paper describes some considerations to be taken in the modular approach to system design. Effective use of modular design can extend the usefulness of equipment and delay its obsolescence. Examples from the manufacture of SLT, IBM's hybrid integrated circuits, are given.

**New Machines for the Reading of Handwritten Symbols and Printed Characters**, H. van Steenis, *Informatie* 9, No. 5, 91-100 (May 1967). An introduction to the IBM 1287 and IBM 1975 with special emphasis on the character recognition aspects and technologies of both systems are given.

**A Note on State Minimization of Asynchronous Sequential Functions**, M. C. Paull and G. Waldbaum, *IEEE Transactions on Electronic Computers* EC-16, No. 1, (February 1967). A procedure is described for minimizing the number of states in an asynchronous sequential function when the restriction exists that the input cannot change while the sequential function is in an unstable state. Furthermore, a procedure is described for minimizing the number of states in a sequential function when the restriction also exists that each output can change at most once during the time required for a transition from one stable state to another.

**Numerical Stability of Difference Equations with Matrix Coefficients**, B. Dejon, *SIAM Journal on Numerical Analysis* 4, No. 1, 119-128 (1967). In this paper we consider the homogeneous difference equation

$$\sum_{j=0}^k \alpha_j y_{n-j} = 0, \quad n = k, k+1, k+2, \dots,$$

with initial values

$$y_j = q_j, \quad j = 0(1)k-1$$

The  $y_j$  are  $d$ -component column vectors, the  $\alpha_j$  are  $d \times d$  matrices independent of  $n$ . We derive algebraic criteria for numerical stability of the difference equation, which is understood in the sense that the solution  $\{y_j\}$  and its difference quotients up to order  $s \in \{0, 1, 2, 3, \dots\}$  depend continuously on the initial values  $\{q_j\}$ . This generalizes the well-known case where  $s = 0$  and the  $\alpha_j$  are diagonal matrices.

**Performance Criteria for the Comparison of Modulation Methods**, R. F. Filipowsky, *1967 SWIEECO Record of Technical Program Papers, Nineteenth Annual Southwestern Conference and Exhibition*, pp. 20-5-1 to 20-5-8 (April 19-21, 1967). A method is presented for the evaluation of the information transmission capability of different modulation systems when compared under well-defined transmission conditions. The method is applicable to analog, sampled, and binary or nonbinary modulation, and it permits the direct comparison of mathematical models with laboratory tests and field tests. This paper reports a further extension of R. W. Sanders' effort to arrive at a suitable normalization of the key parameters used in comparisons of such modulation systems. One novel feature in this paper is the definition of a magnitude called "bit density" which is related to, but not identical with, the  $\alpha$  parameter of Sanders. The bit density is based on the practical magnitude of the transmission rate, which can be easily measured, while the  $\alpha$  parameter is based on the analytical magnitudes of signal and noise entropy that depend on the statistics of the information and, therefore, cannot be easily measured. An expansion of R. W. Sanders' definition of a communications efficiency is explained. It leads, under certain well-defined assumptions, to a system of three performance criteria which are still closely related to the usually applied signal-to-noise ratio and to the transmission rate, yet which permit a normalization for the comparison of the many modulation and coding methods which are now in use. A graphical presentation, the utility chart, will permit the practical engineer to make instant use of these performance criteria.

**PL/I List Processing**, H. W. Lawson, Jr., *Communications of the ACM* 10, No. 6, 358-367 (June 1967). The concepts of list processing have been introduced into the PL/I language. With these new facilities, it is possible to write PL/I procedures that operate on simple and complex data list organizations. Most list-processing languages have suffered from their inability to deal directly with complex data structures and/or from their inability to perform the complete range of programming language operations upon the data list structures. These two problems have been eliminated in the list-processing facilities of PL/I. The basic concepts of list processing and the philosophy of the PL/I language extensions are discussed. Several detailed list-processing examples are also provided.

**Reflections on the Design of a CAI Operating System**, E. N. Adams, *AFIPS Conference Proceedings*, Spring Joint Computer Conference, pp. 419-424 (1967). A time-shared general-purpose computer programmed to operate in conversational mode may be used as a vehicle for computer aided instruction (CAI). For the CAI application certain functions of the system program are especially important, in particular those affecting flexibility of display, system response time, ease with which complex programs may be prepared, proofed, and revised, and the versatility and efficiency of transaction recording and analysis. Some qualitative generalizations about user programs and operational requirements have been suggested by experience with the IBM 1440 Coursewriter, the experimental compiler and operating system for the IBM 7010-1440, and the IBM 1500 Coursewriter II. A few of these will be discussed in terms of their implications for the monitor, service programs, and utilities of the system. Particular attention will be given to optimizing allocations of storage among user and system functions for the kind of user programs we have seen. Some discussion will also be given of entry and compiling, logging and analysis of transaction data, and avoiding or recovering from system malfunctions.

**Statistical Discrimination of the Synonymy/Antonymy Relationship Between Words**, P. A. W. Lewis, P. B. Baxendale, and J. L. Bennett, *Journal of the Association for Computing Machinery* 14, No. 1, 20-44 (January 1967). A basic hypothesis is stated about the contextual and co-occurrence properties of synonymous words. On the basis of this hypothesis, several statistics are derived for use in discriminating between pairs of words which are synonymous and pairs of words which are nonsynonymous. The discriminating power of these statistics is tested on a corpus consisting of titles of physics theses. The tests indicate that two of the derived statistics have relatively high discriminating power. The results are interpreted and the possibility of obtaining better discriminating power is discussed.

**String Similarity and Misspellings**, C. N. Alberga, *Communications of the ACM* 10, No. 5, 302-313 (May 1967). The problem of programming a computer to determine whether or not a string of characters is a misspelling of a given word was considered. A number of algorithms were evaluated—some proposed by other writers, some by the author. These techniques were tested on a collection of misspellings made by students at various grade levels. While many of the methods were clearly unsatisfactory, some gave as few as 2.1 percent incorrect determinations.

**A Study of Remote Industrial Training via Computer-Assisted Instruction**, H. A. Schwartz and H. S. Long, *Journal of Applied Psychology* 51, 11-16 (February 1967). During the latter half of 1965, several field engineers received their required training in new computer technology through remote computer-assisted instruction (CAI). Students at terminals located in 4 major cities communicated, through Teleprocessing facilities, with a computer system located centrally. Students' examination scores, course completion times, and attitudes were compared with those of other students who received the material through self-study texts in use at the time. CAI students scored lower on 1 part of the examination, but completed the course in considerably less time than the self-study students. Attitude scores were somewhat equivocal. Students who had been exposed to both CAI and self-study texts indicated a strong preference for the former. When compared to a "regular classroom" type of presentation, however, the self-study students rated their method slightly higher than did the CAI students. CAI students' attitudes appear to be related to the availability of assistance when course material problems are encountered. Additional findings from locally trained CAI students are presented in support of this interpretation.

**Subresultants and Reduced Polynomial Remainder Sequences**, G. E. Collins,\* *Journal of the Association for Computing Machinery* 14, No. 1, 128-142 (January 1967). Let  $\mathcal{g}$  be an integral domain,  $\mathcal{P}(\mathcal{g})$  the integral domain of polynomials over  $\mathcal{g}$ . Let  $P, Q \in \mathcal{P}(\mathcal{g})$  with  $m = \deg(P) \geq n = \deg(Q) > 0$ . Let  $M$  be the matrix whose determinant defines the resultant of  $P$  and  $Q$ . Let  $M_{ij}$  be the submatrix of  $M$  obtained by deleting the last  $j$  rows of  $P$  coefficients, the last  $j$  rows of  $Q$  coefficients and the last  $2j + 1$  columns, excepting column  $m + n - i - j$  ( $0 \leq i \leq j < n$ ). The polynomial  $R_j(x) = \sum_{i=0}^j \det(M_{ij})x^i$  is the  $j$ -th subresultant of  $P$  and  $Q$ ,  $R_0$  being the resultant. If  $b = \mathcal{L}(Q)$ , the leading coefficient of  $Q$ , there exist uniquely  $R, S \in \mathcal{P}(\mathcal{g})$  such that  $b^{m-n+1}P = QS + R$  with  $\deg(R) < n$ ; define  $\overline{\mathcal{R}}(P, Q) = R$ . Define  $P_i \in \mathcal{P}(\mathcal{F})$ ,  $\mathcal{F}$  the quotient field of  $\mathcal{g}$ , inductively:  $P_1 = P, P_2 = Q, P_3 = \overline{\mathcal{R}}(P_1, P_2), P_{i+2} = \overline{\mathcal{R}}(P_i, P_{i+1})/c_i^{\delta_i-1}$  for  $i \geq 2$  and  $n_{i+1} > 0$ , where  $c_i = \mathcal{L}(P_i), n_i = \deg(P_i)$  and  $\delta_i = n_i - n_{i+1} \cdot P_1, P_2, \dots, P_k$ , for  $k \geq 3$ , is called a reduced polynomial remainder sequence. Some of the main results are: (1)  $P_i \in \mathcal{P}(\mathcal{g})$  for  $1 \leq i \leq k$ ; (2)  $P_k = \pm A_k R_{n_{k-1}}$ , where  $A_k = \prod_{i=2}^{k-2} c_i^{\delta_i-1} (\delta_{i-1})$ ; (3)  $c_k^{\delta_k-1} P_k = \pm A_{k+1} R_{n_k}$ ; (4)  $R_j = 0$  for  $n_k < j < n_{k-1} - 1$ . Taking  $\mathcal{g}$  to be the integers  $I$ , or  $\mathcal{P}^r(I)$ , these results provide new algorithms for computing resultants or greatest common divisors of univariate or multivariate polynomials. Theoretical analysis and extensive testing on a high-speed computer show the new g.c.d. algorithm to be faster than known algorithms by a large factor. When applied to bivariate polynomials, for example, this factor grows rapidly with the degree and exceeds 100 in practical cases.

\* Presently at the University of Wisconsin, Madison, Wisconsin.

**A Time-Sharing System for Business Operations**, G. F. Duffy, *Systems & Procedures Journal* 18, No. 3, 20-24 (May-June 1967). Administrative terminal systems have been demonstrated to be successful in such operations as those that: require file maintenance on a real-time basis at frequent intervals; require information retrieval on a real-time basis; require accurate hard copy, either on or off-line; require minimum turn-around time between entry and output.

**Tool Order Expediting and Control**, F. T. Stussy and J. M. Dunn, *ASTME Annual Engineering Conference*, Preprint MS67-124 (April 24-28, 1967). A system has been developed that effectively controls tool orders from the time the initial tool order is made until the tool is completed and delivered. This expediting and control technique uses data processing cards, standard machine accounting equipment and computer tape sorts. The original tool order is printed and punched in a card. This becomes the master control card which is used for simple computer programs to generate expediting reports, letters to vendors, accounting information, labor claim cards for inplant personnel, and weekly progress report data. The tool order control system provides an organized direct contact with tool vendors to obtain the status of tool orders in their shop and the expected dates. In addition, it provides an effective feedback of information to the engineer who originated the tool request and the production control group. Two people maintain the system. They have all the up-to-date information on the hundreds of tools in the various phases of design and fabrication.

The following papers are from a special series which appeared in the July 1967 issue of the *IBM Journal of Research and Development*.

**Computer Control of a Paper Machine—an Application of Linear Stochastic Control Theory**, K. J. Astrom,\* *IBM Journal of Research and Development* 11, No. 4, 389-405 (July 1967). This paper describes an attempt to apply linear optimal control theory to computer control of an industrial process. The applicability of the theory is discussed. Particular attention is given to the problem of obtaining a mathematical model of process dynamics and disturbances. Results of actual measurements as well as results from on-line control experience are presented.

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**A Computer-Operated Manufacturing and Test System**, J. E. Stuehler and R. V. Watkins, *IBM Journal of Research and Development* 11, No. 4, 452-460 (July 1967). A computer system utilizing duplexed central processors and three types of satellite computer terminals has been developed for the control of manufacturing processes and product tests. The system was designed for use in a plant producing data processing equipment. The prime objectives of the project were (1) to reduce the time required for the plant to accept new products or product changes, (2) to reduce control and test equipment costs by using standard control and test equipment, and (3) to improve product quality through more uniform processes and tests. The objectives were met by using small, stored-program computer terminals to control individual tests and processes. These terminals are connected to a central computer system which assists with data processing and serves as an input/output device for the terminals. The design and implementation of the system occurred in two phases. During the first phase the stored-program test terminal, a high-speed data transmission system, and a multiplexor were designed and installed on an IBM 1460 Data Processing System. A programming system was developed concurrently with the hardware. This system included a real-time monitor and application and utility programs which were written for operation in a time-shared environment. During the second phase of development a process control terminal, a data acquisition terminal, and a new multiplexor for duplexing two 1460 Data Processing Systems were designed and implemented. Also developed were shared file control units, an improved transmission system, and programming designed to effectively utilize the power of two computer systems on a real-time basis.

**Design Principles for Sampled-Data Systems with Application to Attitude Control of a Large, Flexible Booster**, L. A. Gilson, E. F. Harrold, and F. G. Kilmer, *IBM Journal of Research and Development* 11, No. 4, 442-451 (July 1967). This paper reviews  $Z$ -transform and  $W$ -transform theory and discusses in detail its application to dynamic compensation of linear sampled-data control systems. Both sampled-data (digital) and continuous-data (analog) compensator synthesis methods are discussed. With respect to digital compensator design,  $w$ -plane closed-loop pole positions are related to time response characteristics, analogous to the well known relationships between  $s$ -plane pole positions and time response parameters for continuous-data systems. An example is given which illustrates the design technique wherein time and frequency response characteristics are compared. A digital stabilization filter is derived for the attitude control system of a missile typical of the Saturn class.

**A Generalized Legendre-Clebsch Condition for the Singular Cases of Optimal Control**, H. M. Robbins, *IBM Journal of Research and Development* 11, No. 4, 361-372 (July 1967). For certain optimal control problems, some of the extremal trajectories generated by simultaneous solution of the state and adjoint equations may include arcs of a special character, called "singular" arcs. The optimality of singular arcs has been the subject of considerable uncertainty, since the classical criteria are inapplicable or inconclusive. This uncertainty has recently been reduced by the discovery of additional necessary conditions for the optimality of singular arcs. The principal result of this paper is a general statement and proof of these conditions, in the form of a "generalized Legendre-Clebsch condition" which reduces to the classical Legendre-Clebsch condition when applied to nonsingular arcs, and gives additional necessary conditions when applied to singular arcs. Other results include a classification of the possible singular arcs, a useful extension of the conventional optimal-control formalism (by the introduction of "generalized Hamiltonians" and "generalized control transformations"), and some interesting variational formulae.

**On-Line Identification of Process Dynamics**, E. B. Dahlin, *IBM Journal of Research and Development* 11, No. 4, 406-426 (July 1967). Two methods similar to regression analysis are applied to the estimation of parameters in a dynamic process equation. One method uses a residual based directly on the differential equation for the process model. The second method forms the residual from the integro-differential equations derived by integration of the original differential equation with respect to time. The two methods are shown to be complementary in their sensitivity to process and measurement disturbances as well as to errors in the estimate of the process variable reference values. Certain parameters that are very sensitive with one method are shown to be much less sensitive with the other method. A combined method is developed which utilizes each one of the constituent techniques to estimate the parameter for which it has the highest accuracy. This not only permits identification with higher overall parameter accuracy, but also under many practical circumstances gives a convergent solution when one of the constituent methods would give a diverging solution having no practical value for updating control coefficients in an adaptive controller. It is shown that the estimate of the magnitude of a pole in a transfer function can be significantly improved by prefiltering the process input and output data with the same lowpass filter. The paper presents a theoretical and experimental evaluation of the identification methods. Formulas are derived for the variances of the parameters permitting an estimation of the parameter accuracy in a particular test. The test data used has been collected from different control loops on paper machines. The disturbance level on some of the variables is very strong and a test signal-to-noise ratio as low as 0.5 can be encountered. The method is currently used in routine operation in adaptive paper machine control. It is in use on four control loops and has been tested on several other loops as well.



**Rapid Computation of Optimal Trajectories**, K. R. Brown, Jr. and G. W. Johnson, *IBM Journal of Research and Development* 11, No. 4, 373-382 (July 1967). A generalized "indirect" method of solving two-point boundary value problems is discussed in application to the problem of computing optimal trajectories in a vacuum. Improved numerical techniques make the method extremely fast when a good initial estimate of the solution is available, but it also converges, more slowly, from initial estimates that are far from the solution. Transversality conditions are combined with final-value constraints enabling the method to solve directly problems defined by constraints on arbitrary functions of final state. Section 1 defines the differential equations and initial and terminal conditions for optimal rocket trajectories in a central gravitational field. The differential equations are given a particularly simple form and transversality conditions are formulated analytically for typical orbital injection missions. Section 2 defines efficient numerical procedures for solving the initial value problem of optimal trajectories and so reduces the boundary value problem to a multidimensional zero-finding problem. Section 3 describes the generalized version of Newton's method used to solve this multidimensional zero-finding problem. Section 4 summarizes the results of an IBM 7094 implementation, giving execution times and convergence properties.

**Sensitivity Constrained Optimal Control Synthesis**, W. J. Budurka, *IBM Journal of Research and Development* 11, No. 4, 427-435 (July 1967). A constraint on the sensitivity to plant parameter uncertainties is introduced in the synthesis of optimal linear controls by adding a sensitivity function to a quadratic form performance criterion. The synthesis is carried out in the frequency domain using both conventional and Bode sensitivity functions; open-loop and closed-loop controls are of necessity treated separately because the sensitivity functions are different for each case. The optimal control based on the closed-loop Bode sensitivity is shown to satisfy a scalar Wiener-Hopf equation; the same type of equation is also satisfied by the optimal control based on the open-loop conventional sensitivity, but the response characteristics are different. Using the conventional closed-loop sensitivity, the optimal control is shown to satisfy an equation analogous to the Wiener-Hopf equation, but which is more difficult to solve because the control enters quadratically rather than linearly. Examples illustrate the application of derived results.

**Some Results in Two-Point Boundary Value Problems**, S. M. Roberts and J. S. Shipman, *IBM Journal of Research and Development* 11, No. 4, 383-388 (July 1967). Two new results in two-point boundary value problems are presented. The first is a modified method of adjoints which, under certain circumstances, will solve numerically two-point boundary value problems faster than the standard method of adjoints. The second result shows that Friedrichs' solution of the operator equation  $P(x) = 0$  is really the modified Newton method. Kantorovich's sufficiency conditions for the convergence of the modified Newton's method are compared with Friedrichs' sufficiency conditions; it appears that, for most applications, the former conditions allow more leeway.

**Synchronization of Traffic Signals in Grid Networks**, A. Chang, *IBM Journal of Research and Development* 11, No. 4, 436-441 (July 1967). A method of synchronizing traffic signals interconnected in an arbitrary network is presented. The procedure consists of using a simplified mathematical model for traffic to relate the vehicular delay within the network to the signal parameters and then searching over these parameters to minimize the delay. The technique has been used to synchronize traffic signals in San Jose, California and has yielded a ten percent reduction in the average delay per car in comparison with the signal settings determined by the city traffic department with conventional engineering methods.