UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE M22 - 241

By C. W. Gear

TITLE — Eigenvalues and Eigenvectors of a Symmetric Matrix Using the Methods of Givens and Wilkinson. (SADOI Only)

TYPE — Complete Program

NUMBER OF WORDS — 1054 + data storage

MAXIMUM SIZE OF MATRIX — 65 x 65

TIME — Time for all roots and vectors is a maximum of $.096n + .00\ 764n^2 + .000\ 213n^3$ mins. + read in time. Where n is the order of the matrix. If only p values and $q \geq p$ vectors are found the time is approximately

$$p[.096 + .00237\ n] + q\ .0.0014\ n + .00\ 287\ n^2 + .000\ 213\ n^3 \text{ mins.}$$

ACCURACY — If the eigenvalues are distinct, then the vectors are good to ten places or better. The apparent accuracy will increase with n as the vectors are normalised [to length 1], and hence as n increases the relative size of each decreases. If any eigenvalues are close or equal, then the eigenvectors will not be orthogonal for those close roots, but in all cases tried, appear to span the subspace generated by the actual eigenvectors. For further details see the description.

PROGRAMMED CHECKS

FF 03F	in location 209 (sexadecimal)	Incorrect no. of elements in matrix.
FF 03L	173	Drum transfer error
FF 040	213 or 21F or 22F	Sum of squares of elements in matrix exceeds 1.
FF 041	183	Sequence of 0's, 1's, and 2's specifying which roots are to be found is in error. (see input)

FF 042	1SK	Drum transfer error
FF 043	300	Input of master routine in error

INPUT

(Size of matrix must not exceed 65)

The master routine M22 is input in the usual fashion. If no input error occurs it will stop on 34 012 (sexadecimal). If an input error occurs it will stop on FF 043. In the former case, the data punched as below should be input with the black switch.

The elements of the matrix should appear first, punched in the following order:

$$a_{11}\ a_{12}\ a_{13} \cdots\cdots\cdots a_{1n}\ N$$
$$a_{22}\ a_{23} \cdots\cdots\cdots a_{2n}\ N$$
$$a_{33} \cdots\cdots\cdots a_{3n}\ N$$
$$\cdots\cdots\cdots$$
$$\cdots\cdots\cdots$$
$$a_{nn}\ N$$

After each row of the matrix an "N" or any of the characters "J", "F", or "L" must appear. The elements must be punched as a sign followed by up to 12 decimal digits. The sum of the squares of <u>all</u> of the elements of the matrix <u>must not</u> exceed 1, or the machine will stop. Following the matrix a sequence of 0's, 1's, and 2's must be punched, n in number, where n is the order of the matrix. They have the following significance:

If the eigenvalues of the matrix are ordered algebraically, the largest being the first, then if the kth digit is:

- 0 Neither the kth root nor its corresponding vector will be calculated.
- 1 The kth eigenvalue only will be calculated.
- 2 Both the kth eigenvector and value will be calculated.

Following this two sexadecimal characters must be punched, each between 2 and N, the first specifies the number of decimal digits punched for the eigenvalues, the second those of the vectors.

It will compute the required results and finally stop on 34 015 on the R.H.S. of location OSN. A new matrix may be read in with the black switch.

Example: A tape might be punched as follows:

```
+2 + +4 -2 N  +2 -2 +4 N  +2 + N +2 N  (Matrix)
2 0 2 1  3 5              (Sequence and print digits)
```

OUTPUT

For the above example the format would be as follows:

```
+000000000007   (This is the difference of the sum of
                 squares of the elements of the matrix
                 before and after rotation. If this is
  1   +800       unduly large, there may have been an
                 arithmetic error)
+50000
-50000
+50000
-50000
  3   +000
-50000
-50000
+50000
+50000
  4   -400
```

The first eigenvalue and vector were asked for, as were the third. The fourth eigenvalue only was requested. The roots are numbered. The eigenvectors are normalized.

METHOD

The method for finding eigenvalues is the same as in M20, except that the transformation matrix in the reduction to Jacobi form is calculated as well.

If A is the original matrix, and J the Jacobi form, Q is defined by $Q' A Q = J$.

If y is an eigenvector of the matrix J then $x = Qy$ is easily seen to be an eigenvector of A.

If $[\lambda_k]$ are the roots of $|J - \lambda I| = 0$ and if λ^*_k is the machine approximation to the root λ_k, then $(J - \lambda^*_k I)$ is nearly singular. If b is any vector, say $b = \sum_1^n b_i y_i$ where the y_i are the eigenvectors of the matrix J, then the solution of $(J - \lambda^*_k I)y = b$ .....(1)

is $y = (J - \lambda^*_k I)^{-1} b = \sum_{i=1}^{n} \frac{b_i y_i}{\lambda_i - \lambda^*_k}$

Providing that $\frac{b_k}{\lambda_k - \lambda^*_k}$ dominates all other coefficients, then y is a good approximation to y_k. This will happen if b_k is not small, unless there should be two or more close roots. In this case, the apparent eigenvector will be in the space spanned by the eigenvectors corresponding to the adjacent roots. In this method it has been found that with roots different only by 2^{-39} appreciably different vectors are found. They may not be orthogonal, but they define the space within which the true vectors lie. If two roots are equal in the machine then they are made different by 2^{-39} in order to get different vectors.

The solutions of equation (1) are found by triangularizing the matrix $(J - \lambda_k I)$ and then back substituting. The vector b is such that it consists of all 1's after the triangularization process has been carried out. The solution is then normalized, and premultiplied by Q to get the eigenvector of A.

DATE April 18, 1958 RT:5/20/59
PROGRAMMED BY L. W. Spar
APPROVED BY D. E. Muller

lgr

LOCATION	ORDER		NOTES
	00 15K		
15	J0 154F		
	50 L		
16	26 (Y1)		
	00 11010F		Store routine B on the drum
17	00 300F		
	26 999F		
18	J0 154F		Enter
	50 3L		Store routine A on the drum
19	26 (Y1)		
	00 11311F		
20	00 613F		
	26 314F		
21	50 154F		
	50 6L		Read routine A from the drum
22	26 (Y1)		
	00 11311F		
23	00 613F		
	26 314F		
	00 27K		
27	7L 4095F		Constants
	LL 4095F		
28	00 F		
	00 79F		
29	85 11F		
	00 2560F		
30	86 11F		
	00 2560F		
31	00 F		
	00 1F		
32	00 F		
	00 65F		

LOCATION	ORDER	NOTES
33	00 F	
	00 F	
34	00 F	
	00 S3	
35	00 F	
	00 F	
36	00 1F	
	00 F	
37	00 F	
	00 F	
38	00 F	
	00 F	
39	00 F	
	00 F	
40	00 F	
	00 F	
41	80 F	
	00 F	
42	00 F	
	00 765F	
43	00 F	
	00 893F	
44	00 F	
	00 64F	
45	5K 2087F	
	99 2557F	
46	00 F	
	00 633F	
47	00 F	
	00 568F	
48	00 F	
	00 40F	
	(P16)00K	Print routine
	(R1)00K	Square root routine

LOCATION	ORDER	NOTES PAGE 3
	(Y1)00K	Drum transfer routine
	00 154K	Routine B
154	41 450F	
	F5 450F	[Calculation of eigenvectors]
155	40 450F	
	F0 34F	
156	32 172F	
	L5 450F	
157	L4 46F	
	42 163F	
158	L4 32F	
	42 165F	
159	L4 399F	
	42 167F	
160	L4 32F	
	42 168F	
161	42 169F	
	L4 32F	
162	42 170F	
	L0 31F	
163	42 171F	
	L5 F	
164	10 1F	
	40 F	
165	22 165F	
	L5 F	
166	10 1F	
	F4 F	
167	10 1F	
	40 F	
168	22 168F	
	L5 F	

LOCATION	ORDER	NOTES
169	10 1F	
	40 F	
170	22 170F	
	L5 F	
171	10 1F	
	40 F	
172	22 154F	
	F5 171F	
173	42 186F	
	41 455F	
174	L5 34F	
	F4 349F	
175	42 181F	
	L1 34F	
176	L4 31F	
	50 40F	
177	10 1F	
	S4 502F	
178	00 1F	
	L4 35F	
179	40 35F	
	50 F	
180	L5 9F	
	46 208F	
181	46 408F	
	41 F	
182	L5 10F	
	46 435F	
183	L5 34F	
	L4 46F	
184	42 445F	
	42 446F	
185	50 F	
	L5 31F	

LOCATION	ORDER	NOTES PAGE 5	
186	40 451F		
	41 F		
187	F5 455F		
	40 455F		
188	F0 34F		
	34 21F		
189	L5 455F		
	L4 47F		
190	42 192F		
	L4 443F		
191	42 195F		
	42 200F		
192	L5 31F		
	L0 F		
193	32 201F		
	L5 451F		
194	L4 443F		
	L4 47F		
195	42 196F		
	L5 F		
196	40 12F		
	L0 F		
197	36 198F		
	22 210F		
198	40 1F		
	F1 1F		
199	L4 12F		
	40 12F		
200	22 200F		
	40 F		
201	22 210F		
	L0 31F		
202	36 187F		
	L5 195F		

LOCATION	ORDER	NOTES	PAGE 6
203	42 205F		
	L5 455F		
204	J2 3F		
	50 204F		
205	26 (P16)		
	L5 12F		
206	00 1F		
	40 F		
207	92 967F		
	L5 F		
208	50 F		
	50 208F		
209	26 (P16)		
	92 139F		
210	26 187F		
	L5 35F		
211	40 453F		
	41 450F		
212	L5 894F		
	L0 12F		
213	40 634F		
	L5 959F		
214	40 699F		
	41 764F		
215	F5 450F		
	40 450F		
216	L4 46F		
	42 220F		
217	F4 399F		
	42 257F		
218	L4 428F		
	42 221F		
219	L5 450F		
	L0 34F		

LOCATION	ORDER	NOTES	PAGE 7
220	36 264F		
	L5 F		
221	40 1F		
	L5 F		
222	40 2F		
	L7 1F		
223	L2 2F		
	32 244F		
224	L5 450F		
	L4 46F		
225	42 240F		
	L4 31F		
226	42 239F		
	L4 44F		
227	42 238F		
	42 241F		
228	L4 31F		
	42 236F		
229	L4 44F		
	42 243F		
230	F4 399F		
	42 232F		
231	L4 32F		
	42 233F		
232	L1 12F		
	L4 F		
233	40 3F		
	50 F		
234	L5 233F		
	42 242F		
235	71 1F		
	66 2F		
236	S5 F		
	40 F		

LOCATION	ORDER	NOTES	PAGE 8
237	50 3F		
	71 1F		
238	66 2F		
	L5 F		
239	S4 F		
	40 F		
240	L5 2F		
	40 F		
241	L5 3F		
	40 F		
242	22 242F		
	L5 F		
243	22 243F		
	40 F		
244	22 257F		
	40 3F		
245	L5 450F		
	F4 46F		
246	42 255F		
	L4 44F		
247	42 252F		
	42 258F		
248	L4 31F		
	42 256F		
249	F4 428F		
	42 253F		
250	L4 32F		
	42 254F		
251	L3 3F		
	32 258F		
252	50 2F		
	71 F		
253	66 1F		
	L5 F		

LOCATION	ORDER	NOTES	PAGE 9
254	S4 F		
	50 F		
255	L0 12F		
	40 F		
256	S5 F		
	40 F		
257	22 257F		
	41 F		
258	26 215F		
	L5 F		
259	40 3F		
	L5 2F		
260	L4 1F		
	40 4F		
261	L3 4F		
	36 263F		
262	L1 3F		
	40 3F		
263	50 3F		
	22 253F		
264	L5 34F		
	L4 46F		
265	42 266F		
	F0 399F		
266	42 268F		
	L5 F		
267	40 457F		
	26 268F		
268	L5 27F		
	40 F		
269	L5 450F		
	L0 31F		
270	40 450F		
	L4 349F		

LOCATION	ORDER	NOTES	PAGE 10
271	42 330F		
	L4 31F		
272	42 283F		
	L4 399F		
273	42 314F		
	42 280F		
274	L4 32F		
	42 282F		
275	L4 32F		
	42 331F		
276	50 F		
	42 287F		
277	F5 283F		
	42 297F		
278	F5 331F		
	42 299F		
279	41 452F		
	L3 450F		
280	36 337F		
	L3 F		
281	36 447F		
	50 F		
282	22 282F		
	50 F		
283	22 283F		
	71 F		
284	40 1F		
	36 290F		
285	L5 457F		
	36 291F		
286	L4 1F		
	36 293F		
287	40 456F		
	L5 F		

LOCATION	ORDER	NOTES	PAGE 11
288	40 1F		
	L3 1F		
289	32 314F		
	22 297F		
290	L5 457F		
	36 292F		
291	L4 1F		
	26 287F		
292	L4 1F		
	36 287F		
293	10 1F		
	L4 41F		
294	40 456F		
	F5 452F		
295	40 452F		
	L5 457F		
296	10 1F		
	40 457F		
297	22 287F		
	50 F		
298	71 1F		
	40 1F		
299	L5 452F		
	L4 F		
300	42 302F		
	40 2F		
301	L3 2F		
	32 303F		
302	L5 1F		
	10 F		
303	40 1F		
	L5 1F		
304	36 307F		
	L5 456F		

LOCATION	ORDER	NOTES	PAGE 12
305	36 308F		
	L4 1F		
306	32 310F		
	22 308F		
307	L5 456F		
	32 309F		
308	L4 1F		
	40 456F		
309	22 314F		
	L4 1F		
310	32 308F		
	10 1F		
311	L4 41F		
	40 456F		
312	F5 452F		
	40 452F		
313	F5 457F		
	10 1F		
314	40 457F		
	L5 F		
315	40 1F		
	L7 1F		
316	L2 456F		
	50 40F		
317	32 327F		
	41 3F		
318	L5 456F		
	36 326F		
319	L1 1F		
	40 1F		
320	F5 3F		
	42 3F		
321	L7 456F		
	10 1F		

LOCATION	ORDER	NOTES	PAGE 13
322	40 456F		
	L2 1F		
323	L0 31F		
	36 320F		
324	L5 3F		
	42 326F		
325	L4 452F		
	40 452F		
326	L5 457F		
	10 1F		
327	40 457F		
	L7 456F		
328	L2 1F		
	32 332F		
329	L5 456F		
	66 1F		
330	S5 F		
	40 F		
331	L5 452F		
	40 F		
332	26 269F		
	L5 456F		
333	L4 1F		
	40 3F		
334	L3 3F		
	36 336F		
335	L5 27F		
	22 330F		
336	L1 27F		
	22 330F		
337	L5 34F		
	40 450F		
338	41 10F		
	41 11F		

LOCATION	ORDER	NOTES	
339	41 452F		
	L5 450F		
340	L4 349F		
	42 350F		
341	F4 443F		
	42 342F		
342	50 40F		
	L5 F		
343	00 1F		
	42 348F		
344	40 1F		
	L1 1F		
345	L4 449F		
	32 346F		
346	27 349F		
	L3 1F		
347	32 350F		
	50 11F		
348	L5 10F		
	10 F		
349	40 10F		
	S5 502F		
350	40 11F		
	L5 F		
351	40 1F		
	50 1F		
352	75 1F		
	40 F		
353	L5 452F		
	42 357F		
354	L1 452F		
	L4 449F		
355	36 356F		
	27 358F		

LOCATION	ORDER	NOTES	PAGE 15
356	L3 452F		
	32 358F		
357	L5 F		
	10 F		
358	40 F		
	L5 11F		
359	S4 F		
	36 362F		
360	L4 41F		
	40 11F		
361	F5 F		
	40 F		
362	L5 F		
	L4 10F		
363	40 10F		
	36 368F		
364	50 11F		
	10 1F		
365	L4 41F		
	40 10F		
366	S5 F		
	40 11F		
367	F5 452F		
	40 452F		
368	L5 450F		
	L0 31F		
369	40 450F		
	L0 31F		
370	32 339F		
	50 40F		
371	L5 452F		
	10 1F		
372	40 452F		
	S5 F		

LOCATION	ORDER	NOTES
373	L0 31F	
	32 397F	
374	L5 12F	
	40 4F	
375	L5 11F	
	40 F	
376	L5 10F	
	50 376F	
377	26 (R1)	
	40 10F	
378	F5 450F	
	40 450F	
379	L4 177F	
	42 381F	
380	42 392F	
	F4 443F	
381	42 390F	
	L5 F	
382	40 F	
	L3 452F	
383	36 395F	
	50 40F	
384	L5 452F	
	42 386F	
385	L0 48F	
	32 390F	
386	L5 F	
	10 F	
387	40 F	
	L7 F	
388	L2 10F	
	32 396F	
389	L5 F	
	50 40F	

LOCATION	ORDER	NOTES	PAGE 17
390	66 10F		
	L5 F		
391	L4 452F		
	40 452F		
392	S5 F		
	40 F		
393	F1 450F		
	L4 34F		
394	36 378F		
	26 402F		
395	50 40F		
	L5 F		
396	26 387F		
	50 27F		
397	22 390F		
	L5 10F		
398	50 11F		
	10 1F		
399	40 10F		
	S5 130F		
400	40 11F		
	F5 452F		
401	40 452F		
	26 374F		
402	L5 31F		
	40 454F		
403	L5 4F		
	00 1F		
404	40 4F		
	L5 455F		
405	J2 3F		
	50 405F		
406	26 (P16)		
	92 967F		

LOCATION	ORDER	NOTES
407	L5 4F	
	22 408F	
408	50 F	
	50 408F	
409	26 (P16)	
	92 135F	
410	L5 503F	
	22 435F	
411	F5 454F	
	40 454F	
412	L4 444F	
	40 415F	
413	F5 46F	
	42 416F	
414	42 420F	
	26 415F	
415	00 F	
	00 F	
416	22 416F	
	40 F	
417	L5 415F	
	L4 32F	
418	40 415F	
	F5 416F	
419	42 416F	
	L0 445F	
420	36 415F	
	L1 F	
421	L4 453F	
	40 453F	
422	F5 420F	
	42 420F	
423	L0 446F	
	32 420F	

LOCATION	ORDER	NOTES PAGE 19	
424	L5 31F		
	40 450F		
425	41 10F		
	50 40F		
426	L5 450F		
	L4 46F		
427	42 429F		
	L0 399F		
428	42 430F		
	S5 194F		
429	22 429F		
	50 F		
430	22 430F		
	74 F		
431	L4 10F		
	40 10F		
432	F5 450F		
	40 450F		
433	L0 34F		
	32 434F		
434	26 426F		
	L5 10F		
435	50 F		
	50 435F		
436	26 (P16)		
	92 131F		
437	92 515F		
	L5 454F		
438	L0 34F		
	32 439F		
439	26 411F		
	L5 455F		
440	40 451F		
	92 135F		

LOCATION	ORDER	NOTES	PAGE 20
441	L3 453F		
	36 443F		
442	FF 66F		
	FF 66F		
443	26 187F		
	00 260F		
444	85 11F		
	00 6783F		
445	K2 416F		
	40 F		
446	S6 415F		
	L1 F		
447	L5 48F		
	40 452F		
448	L5 27F		
	22 330F		
449	00 F		
	00 60F		
450	00 F		
	00 F		
451	00 F		
	00 F		
452	00 F		
	00 F		
453	00 F		
	00 F		
454	00 F		
	00 F		
455	00 F		
	00 F		
456	00 F		
	00 F		
457	00 F		
	00 F		

LOCATION	ORDER	NOTES PAGE 21
	26 15N	Interlude to store B on drum.
	00 24K	
24	50 154F	
	50 L	Read routine B from drum
25	26 (Y1)	
	00 11010F	
26	00 300F	
	26 154F	
	00 154K	
154	41 451F	J =
	F5 451F	J → J + 1
155	40 451F	
	F0 34F	J = N + 1 ?
156	36 24F	
	L5 451F	
157	L4 197F	
	42 158F	Are we to find this (J) value?
158	32 158F	
	L1 F	[A_2 - calculate eigenvalues from
159	32 154F	reduced form]
	L5 451F	
160	L4 46F	
	42 163F	
161	42 167F	
	L4 32F	
162	42 165F	
	42 169F	
163	L5 313F	
	L4 F	
164	10 1F	
	40 F	
165	50 296F	
	L1 F	

LOCATION	ORDER	NOTES PAGE 22
166	10 1F	
	L4 F	$L_J - U_J + 2^{-38} \geq 0$?
167	32 154F	
	L5 F	
168	10 1F	
	40 F	
169	50 297F	
	F5 F	$u = (U_J + L_J)/2.$
170	10 1F	
	L4 F	
171	40 38F	
	41 450F	$I = 0$
172	41 452F	
	41 3F	$m_{-1} = 0$
173	41 8F	$V_0 = 0$
	L5 27F	
174	40 4F	
	F5 450F	$m_0 = 1 - 2^{-39}$
175	42 450F	$I \rightarrow I + 1$
	F0 34F	
176	32 240F	$I = N + 1$?
	L5 450F	
177	L4 192F	
	42 188F	
178	L4 32F	
	42 195F	Set I dependent addresses.
179	42 212F	
	42 233F	
180	L4 32F	
	42 183F	
181	L1 38F	
	40 7F	
182	41 6F	
	L5 255F	Link for scaling routine.

LOCATION	ORDER	NOTES PAGE 23
183	40 298F	Scale a_i
	L5 F	
184	22 264F	
	L3 3F	$m_{i-2} = 0$?
185	32 223F	
	L3 4F	$m_{i-1} = 0$?
186	32 233F	
	L5 256F	Link for scaling routine.
187	40 298F	
	L5 8F	$\Delta_i = V_{i-1} + \delta_{i-\sum i-1}$
188	L4 2F	
	L0 F	
189	40 1F	
	32 210F	$\Delta_i \geq 0$?
190	L5 254F	
	42 200F	$\pi_1 \rightarrow \pi_j$
191	50 F	
	75 4F	$p_i \, m_{i-1}$
192	40 7F	
	S5 763F	
193	40 11F	
	L1 1F	
194	42 196F	
	L0 48F	
195	32 209F	
	50 F	$\Delta \geq 40$?
196	75 3F	
	10 F	$q_{i-1} \cdot m_{i-2}$
197	40 5F	
	S5 568F	Subtract
198	40 6F	
	L5 2F	
199	40 8F	
	L5 11F	

LOCATION	ORDER	NOTES PAGE 24
200	26 257F	Scale to m_i and Σ_i
	26 F	$\rightarrow \pi_j$
201	L5 2F	
	L4 8F	
202	40 8F	
	L5 4F	$V_i = \Sigma_i + \delta_i$
203	40 3F	
	L5 F	
204	40 4F	
	L5 3F	$m_{i-1} \geq 0$?
205	36 208F	
	L5 4F	
206	32 174F	$m_i \geq 0$?
	F5 452F	$p \rightarrow p + 1$
207	42 452F	
	22 174F	
208	L5 4F	
	32 206F	$m_i \geq 0$?
209	22 174F	
	41 5F	
210	26 198F	
	42 219F	$\pi_2 \rightarrow \pi_j$
211	L5 221F	
	42 200F	$q_{i-1} \cdot m_{i-2}$
212	50 3F	
	75 F	$p_i m_{i-1}$
213	40 5F	
	S5 F	
214	40 6F	
	50 F	
215	75 4F	
	40 7F	
216	L3 1F	$\Delta = 0$?
	32 220F	

LOCATION	ORDER	NOTES	PAGE 25
217	L4 48F		
	36 219F		$\Delta \geq 40$?
218	22 222F		
	00 F		
219	L5 7F		
	10 F		
220	40 7F		
	L5 2F		
221	40 8F		
	S5 231F		
222	26 257F		Scale to m_i and Σ_i
	41 7F		
223	26 257F		
	L1 4F		$m_{i-1} \geq 0$?
224	36 228F		
	L5 27F		
225	40 3F		
	L5 F		$m_{i-1} = 1 - 2^{-39}$
226	40 4F		
	L5 2F		$m_i = p_i$
227	40 8F		
	22 204F		
228	L1 27F		
	40 3F		$m_{i-1} = -1 + 2^{-39}$
229	L1 F		
	40 4F		$m_i = -p_i$
230	22 204F		
	00 F		
231	L1 1F		
	L4 8F		
232	L4 2F		
	40 8F		
233	22 202F		
	L3 F		$q_{i-1} = 0$?

LOCATION	ORDER	NOTES
234	32 224F	
	L5 3F	$m_{i-2} \geq 0$?
235	32 237F	
	L5 27F	
236	40 4F	
	41 3F	$m_i = 1 - 2^{-39}$
237	22 204F	
	L1 27F	$m_i = -1 + 2^{-39}$
238	26 236F	
	L5 4F	
239	40 3F	
	41 4F	
240	22 204F	
	L5 451F	K = J
241	40 454F	
	L5 454F	
242	L4 46F	
	42 251F	Set K dependent
243	42 253F	addresses.
	L4 32F	
244	42 248F	
	42 246F	
245	L5 452F	
	L0 454F	
246	32 251F	
	L1 F	$-u_K + u \geq 0$?
247	L4 38F	
	36 249F	
248	L5 38F	
	40 F	$U_K = u$
249	F5 454F	
	42 454F	K → K + 1
250	F0 34F	
	36 163F	K = n + 1 ?

LOCATION	ORDER	NOTES PAGE 27
251	22 241F	
	L5 F	$L_K - u \geq 0$?
252	L0 38F	
	36 249F	
253	L5 38F	
	40 F	$L_K = u$.
254	26 249F	
	00 201F	
255	22 184F	
	22 184F	Constants for exit
256	22 200F	from scale routine.
	22 238F	
257	L0 6F	
	32 262F	Form least significant
258	L4 41F	
	40 6F	half and test for
259	L7 5F	
	36 264F	carry. This part
260	L4 7F	
	36 299F	does a double precision
261	L0 31F	
	36 299F	subtraction before
262	22 267F	
	40 6F	scaling of
263	L1 5F	
	22 264F	N(7, A) -N(5, 6)
264	F1 5F	
	40 5F	Scale routine
265	36 302F	
	L5 7F	
266	36 303F	
	L4 5F	Number is made negative
267	36 299F	
	40 F	

LOCATION	ORDER	NOTES
268	41 2F	
	L5 169F	
269	46 296F	
	L3 F	Test for 0.
270	32 298F	
	50 6F	
271	L5 F	
	00 1F	
272	36 282F	
	00 1F	Scale by 2
273	36 283F	until positive
	00 1F	
274	32 284F	
	00 1F	
275	36 286F	
	00 1F	
276	32 287F	
	00 1F	
277	36 289F	
	00 1F	
278	32 290F	
	00 1F	
279	36 292F	
	40 F	
280	F5 265F	
	L4 2F	Increase scale factor by 8
281	42 2F	
	26 271F	
282	40 F	
	27 293F	
283	40 F	
	L5 31F	Increase scale factor by
284	26 293F	appropriate amount
	40 F	
285	F5 31F	
	26 293F	

LOCATION	ORDER	NOTES PAGE 29
286	40 F	
	L5 236F	
287	26 293F	
	40 F	
288	F5 236F	
	26 293F	
289	40 F	
	L5 264F	
290	26 293F	
	40 F	
291	F5 264F	
	26 293F	
292	40 F	
	L5 265F	
293	L4 2F	
	42 2F	Add to scale factor
294	L5 F	
	10 1F	Adjust scaled number to correct
295	L4 41F	sign and ÷ by 2.
	40 F	
296	22 F	
	L1 F	
297	40 F	
	26 298F	
298	22 F	
	22 F	exit.
299	10 1F	
	L4 41F	Overflow adjustment,
300	40 F	
	L1 31F	
301	40 2F	
	26 298F	
302	L5 7F	
	32 304F	

LOCATION	ORDER	NOTES PAGE 30	
303	L4 5F		
	36 306F		
304	22 267F		
	L4 5F		
305	36 306F		
	26 299F		
306	40 F		
	41 2F		
307	L5 165F		
	46 296F		
308	L1 6F		
	32 311F		
309	L4 41F		
	40 6F		
310	F1 F		
	40 F		
311	22 269F		
	L1 F		
312	40 F		
	22 269F		
313	00 F		
	00 2F		
314	41 35F		
	26 508F		
	00 K		
315	L5 29F		
	40 4L		[A_1: reduction of matrix to
316	L4 32F		Jacobi form and read in of
	40 10L		matrix].
317	F5 43F		
	42 5L		
318	F4 32F		
	42 11L		Read out from drum of reduced matrix.

LOCATION	ORDER	NOTES
319	85 11F	
	00 2560F	
320	22 5L	
	40 F	
321	L5 4L	
	F4 32F	
322	40 4L	
	F5 5L	
323	42 5L	
	L0 15L	
324	36 16L	
	26 10L	
325	85 11F	
	00 2560F	
326	22 11L	
	40 F	
327	L5 10L	
	F4 32F	
328	40 10L	
	F5 11L	
329	42 11L	
	26 4L	
330	22 5L	
	40 959F	
331	41 959F	
	41 392L	
332	41 10F	
	41 11F	
333	F5 392L	
	42 392L	
334	F0 34F	
	32 51L	
335	L5 392L	
	L4 42F	

LOCATION	ORDER	NOTES	PAGE 32
336	F0 31F		
	42 47L		
337	L4 32F		
	42 46L		
338	L4 32F		
	42 27L		
339	42 36L		
	42 37L		
340	L4 32F		
	42 28L		
341	42 29L		
	42 30L		
342	41 3F		
	L1 F		
343	L4 33F		
	L0 F		
344	40 33F		
	50 F		
345	22 30L		
	75 F		Sum squares
346	00 1F		
	40 F		
347	S5 F		
	40 1F		
348	L4 11F		
	10 39F		
349	40 2F		
	L7 2F		
350	L4 10F		
	40 10F		
351	S5 F		
	50 F		
352	22 37L		
	74 F		

LOCATION	ORDER	NOTES
353	L4 10F	
	L4 F	
354	40 10F	
	S5 959F	
355	40 11F	
	L3 F	
356	32 48L	
	50 1F	
357	L5 F	
	00 1F	
358	40 F	
	F5 3F	
359	42 3F	
	L5 F	
360	32 42L	
	10 1F	
361	L4 41F	
	40 F	
362	L5 3F	
	40 F	
363	26 18L	
	L3 1F	
364	36 50L	
	22 41L	
365	L5 28F	
	40 3F	
366	23 46L	
	92 139F	
367	L5 37F	
	L0 10F	
368	50 12F	
	50 53L	Print difference of sum of
369	26 (P16)	squares
	92 135F	

LOCATION	ORDER	NOTES
370	L3 33F	
	36 57L	Sum check on drum for A.
371	FF 63F	
	00 568F	
372	41 39F	
	F5 39F	
373	40 39F	
	F0 34F	
374	36 65L	
	L5 39F	
375	L4 64L	
	42 62L	
376	L4 32F	
	42 63L	
377	L1 38F	
	40 634F	
378	L5 38F	
	40 699F	
379	22 57L	
	00 633F	
380	41 39F	
	F5 39F	
381	42 39F	
	F0 34F	
382	32 73L	
	L5 39F	
383	L4 56L	
	42 69L	Read in sequence of 0's, 1's,
384	81 4F	and 2's.
	40 F	
385	40 F	
	L1 F	
386	F4 31F	
	36 73L	

LOCATION	ORDER	NOTES PAGE 35
387	FF 65F	
	FF 65F	
388	22 65L	
	81 4F	
389	00 20F	
	46 9F	Read in digits specifying no.
390	81 4F	of places to print.
	00 20F	
391	46 10F	
	26 154F	
392	40 F	Drum subroutine
	K5 102L	II
393	42 82L	
	L4 31F	
394	42 101L	
	10 20F	
395	42 94L	
	42 98L	
396	42 102L	
	42 107L	
397	22 82L	
	L4 F	
398	42 111L	
	42 112L	
399	42 113L	
	42 114L	
400	S5 F	
	36 88L	
401	L5 77L	
	42 92L	
402	L5 116L	
	22 89L	
403	L5 115L	
	42 92L	

LOCATION	ORDER	NOTES	PAGE 36
404	L5 117L		
	L4 F		
405	50 40F		
	00 6F		
406	L4 F		
	40 93L		
407	40 103L		
	26 F		
408	00 F		
	00 F		
409	32 94L		
	40 F		
410	F5 93L		
	40 93L		
411	F5 94L		
	42 94L		
412	L0 111L		
	36 93L		
413	L5 35F		
	L0 F		
414	40 35F		
	F5 98L		
415	42 98L		
	L0 112L		
416	36 98L		
	26 F		
417	32 102L		
	L5 F		
418	00 F		
	00 F		
419	F5 103L		
	40 103L		
420	F5 102L		
	42 102L		

LOCATION	ORDER	NOTES	PAGE 37
421	L0 113L		
	32 102L		
422	L5 35F		
	L4 F		
423	40 35F		
	F5 107L		
424	42 107L		
	L0 114L		
425	36 107L		
	22 101L		
426	S2 94L		
	40 F		
427	75 35F		
	L0 F		
428	S2 102L		
	L5 F		
429	75 35F		
	L4 F		Constants
430	00 F		
	00 93L		
431	02 384F		
	2N 105F		
432	02 320F		
	2N 105F		
433	40 3F		Drum subroutine
	K5 F		I
434	42 126L		
	10 20F		
435	42 2F		
	36 123L		
436	L5 182L		
	42 148L		
437	F5 181L		
	22 124L		

LOCATION	ORDER	NOTES
438	L5 140L	
	42 148L	
439	L5 181L	
	42 137L	
440	42 143L	
	41 1F	
441	43 139L	
	L5 F	
442	42 1F	
	F5 126L	
443	42 166L	
	L5 2F	
444	L4 1F	
	42 153L	
445	42 167L	
	42 162L	
446	42 177L	
	F5 1F	
447	L0 3F	
	36 149L	
448	L5 3F	
	L0 31F	
449	40 F	
	L4 2F	
450	42 183L	
	42 184L	
451	50 F	
	75 32F	
452	S5 F	
	L4 F	
453	L4 1F	
	40 152L	
454	40 168L	
	50 F	

LOCATION	ORDER	NOTES
455	L5 2F	
	S4 152L	
456	42 158L	
	42 172L	
457	75 32F	
	S5 5F	
458	L4 3F	
	L4 F	
459	L0 31F	
	40 157L	
460	40 173L	
	L5 2F	
461	L4 34F	
	42 185L	
462	42 186L	
	42 187L	
463	42 188L	
	26 F	
464	L5 31F	
	42 139L	
465	L5 148L	
	L4 142L	
466	42 148L	
	22 139L	
467	00 F	
	00 F	
468	32 153L	
	40 F	
469	F5 152L	
	40 152L	
470	F5 153L	
	42 153L	
471	L0 183L	
	36 152L	

LOCATION	ORDER	NOTES	PAGE 40
472	00 F		
	00 F		
473	32 158L		
	40 F		
474	L5 157L		
	L4 32F		
475	40 157L		
	F5 158L		
476	42 158L		
	L0 185L		
477	36 157L		
	L1 F		
478	L4 33F		
	40 33F		
479	F5 162L		
	42 162L		
480	L0 187L		
	32 162L		
481	32 166L		
	26 F		
482	32 167L		
	L5 F		
483	00 F		
	00 F		
484	F5 168L		
	40 168L		
485	F5 167L		
	42 167L		
486	L0 184L		
	32 167L		
487	32 172L		
	L5 F		
488	00 F		
	00 F		

LOCATION	ORDER	NOTES	PAGE 41
489	L5 173L		
	L4 32F		
490	40 173L		
	F5 172L		
491	42 172L		
	L0 186L		
492	32 172L		
	L5 F		
493	L4 33F		
	40 33F		
494	F5 177L		
	42 177L		
495	L0 188L		
	32 177L		
496	22 166L		
	00 29F		
497	00 F		
	00 167L		
498	S2 153L		
	40 F		
499	S2 167L		
	L5 F		
500	S2 158L		
	40 F		
501	S2 172L		
	L5 F		
502	S6 157L		
	L1 F		
503	S2 172L		
	L5 F		
504	L5 198L		
	L4 36F		
505	46 198L		
	10 20F		

LOCATION	ORDER	NOTES PAGE 42
506	42 212L	
	42 213L	
507	42 214L	
	26 198L	
508	L5 208L	Read in begins here.
	46 198L	
509	L5 39L	
	42 212L	Reset routine
510	42 213L	
	42 214L	
511	41 10F	
	41 11F	
512	43 209L	
	41 241L	
513	50 959F	
	50 198L	
514	26 (N12)	
	41 F	Read in 1 row
515	L5 21(N12)	
	26 409L	Find no. of elements read in.
516	42 34F	
	L5 34F	On first row set loc. 34 = to N,
517	L4 408L	on each subsequent row check
	40 238L	to see that N-1, N-2, ... 1
518	L5 236L	elements are read.
	40 201L	
519	26 207L	
	L0 233L	
520	40 F	
	L3 F	
521	36 207L	
	FF 62F	
522	F5 241L	
	42 241L	

LOCATION	ORDER	NOTES PAGE 43
523	J0 959F	
	50 208L	Store row on drum
524	26 118L	using routine I
	00 F	
525	L5 241L	
	42 209L	
526	41 12F	
	41 13F	
527	L5 39F	
	L4 959F	
528	40 39F	
	50 959F	Sum squares.
529	L5 13F	
	74 959F	
530	10 1F	
	32 216L	Test to see if sum exceeds 1.
531	FF 64F	
	40 12F	
532	S5 959F	
	40 13F	
533	L5 239L	
	40 215L	
534	F5 213L	
	42 213L	
535	42 214L	
	42 212L	
536	L0 238L	
	36 212L	
537	L5 11F	
	L4 13F	
538	10 39F	
	40 F	
539	S5 F	
	40 11F	

LOCATION	ORDER	NOTES	PAGE 44
540	L1 F		
	L4 10F		
541	L4 12F		
	32 227L		
542	FF 64F		
	40 10F		
543	L5 240L		
	40 215L		
544	L5 234L		
	L0 31F		
545	40 233L		
	L0 235L		Test for last row
546	36 189L		
	L5 237L		
547	40 201L		
	26 242L		
548	00 F		
	00 F		
549	00 F		
	00 F		
550	LL 4095F		
	L0 1F		
551	22 204L		
	L5 34F		
552	42 34F		
	L5 34F		Constants
553	N0 39F		
	50 F		
554	L4 12F		
	32 216L		
555	10 1F		
	32 216L		
556	00 F		
	00 F		

LOCATION	ORDER	NOTES PAGE 45
557	L5 10F	
	00 1F	
558	36 244L	
	FF 64F	
559	40 37F	
	50 244L	Square root of sum of squares
560	26 (R1)	
	40 38F	
561	26 247L	
	00 F	
562	L5 257L	
	40 248L	
563	87 11F	
	00 6789F	Set Q = I on drum
564	F5 248L	
	40 248L	
565	L0 259L	
	36 248L	
566	L5 258L	
	40 253L	
567	22 252L	
	L5 27F	
568	86 11F	
	00 6789F	
569	L5 253L	
	F4 32F	
570	40 253L	
	L0 260L	
571	32 252L	
	26 261L	
572	87 11F	
	00 6785F	
573	86 11F	
	00 6785F	

LOCATION	ORDER	NOTES PAGE 46
574	07 11F	
	00 10946F	
575	06 11F	
	00 10946F	
576	41 392L	i = 0
	L5 34F	[Begin rotation]
577	L0 31F	
	42 389L	
578	42 282L	Set test constants
	L5 392L	
579	42 279L	
	42 291L	
580	42 377L	
	42 386L	
581	F5 392L	
	40 392L	i → i +1
582	40 392L	
	F5 392L	
583	L0 34F	
	36 L	Test for end of rotation
584	L5 392L	i = n-1 ?
	L4 43F	
585	42 296L	
	L4 31F	
586	42 362L	Set i dependent addresses.
	42 371L	
587	42 324L	
	L4 44F	
588	42 297L	
	42 374L	
589	L4 31F	
	42 327L	
590	42 366L	
	42 370L	

LOCATION	ORDER		NOTES
591	42 292L		
	42 378L		
592	F5 392L		
	40 393L		j = i+1
593	50 894F		
	50 278F		Read out one row of A.
594	26 118L		
	00 F		
595	L5 392L		
	22 281L		
596	50 766F		
	50 281L		Read out one row of Q.
597	26 77L		
	00 F		
598	F5 393L		
	42 391L		
599	L5 393L		
	42 295L		Set j-dependent addresses.
600	42 381L		
	F4 43F		
601	42 364L		
	42 369L		
602	42 326L		
	F4 44F		
603	42 368L		
	42 373L		
604	42 325L		
	F5 393L		
605	50 959F		
	50 290L		
606	26 118L		
	00 F		Read out a row of A
607	L5 33F		
	L4 F		

LOCATION	ORDER	NOTES	PAGE 48
608	40 33F		
	L5 393L		
609	50 830F		
	50 294L		Read out a row of Q
610	26 77L		
	00 F		
611	22 296L		
	L5 F		
612	40 3F		
	L5 F		
613	40 4F		
	L3 4F		
614	32 375L		
	L7 3F		
615	L2 4F		
	36 398L		
616	50 3F		
	L5 4F		
617	40 3F		
	S5 F		
618	40 4F		
	L5 397L		
619	42 317L		
	46 324L		
620	50 40F		
	L5 4F		
621	66 3F		
	S5 F		Calculate C_{ij} and S_{ij}
622	40 5F		
	7J 5F		
623	10 1F		
	40 1F		
624	L3 1F		
	36 407L		

LOCATION	ORDER	NOTES
625	LJ 1F	
	40 1F	
626	50 40F	
	6L 1F	
627	41 F	
	K5 F	
628	36 314L	
	26 407L	
629	S5 F	
	50 314L	
630	26 (R1)	
	40 1F	
631	50 1F	
	7J 5F	
632	40 F	
	40 F	
633	50 F	
	79 F	
634	40 F	
	50 1F	
635	79 1F	
	L4 F	
636	L0 41F	
	50 40F	
637	66 1F	
	S5 F	
638	10 1F	
	L4 1F	
639	40 F	
	L5 F	
640	22 325L	
	L4 F	
641	40 2F	
	L5 F	

LOCATION	ORDER	NOTES PAGE 50
642	22 327L	
	40 F	
643	22 328L	
	L5 392L	Prepare for (i,j) rotation of A
644	40 394L	
	L5 394L	
645	L4 43F	
	42 334L	
646	42 342L	
	42 340L	
647	F4 44F	
	42 336L	
648	42 338L	
	42 341L	
649	50 5F	
	75 F	
650	40 F	
	S5 F	Rotate in (i,j)th plane
651	50 6F	
	74 F	
652	L4 F	
	40 1F	
653	50 5F	
	75 F	
654	40 F	
	S5 F	
655	50 6F	
	70 F	
656	L4 F	
	40 F	
657	L5 1F	
	40 F	
658	F5 394L	Test for finish of (i,j)th
	42 394L	rotation

LOCATION	ORDER	NOTES PAGE 51	
659	F0 34F		
	32 345L		
660	22 329L		
	L5 31F		
661	40 395L		
	L5 42F		
662	L4 395L		
	42 351L		Prepare for (i,j) rotation of Q
663	42 357L		
	42 359L		
664	L4 44F		
	42 353L		
665	42 355L		
	42 358L		
666	50 5F		
	75 F		
667	40 F		
	S5 F		
668	50 6F		
	74 F		
669	L4 F		
	40 1F		
670	50 5F		
	75 F		
671	40 F		
	S5 F		
672	50 6F		
	70 F		Rotate.
673	L4 F		
	40 F		
674	L5 1F		
	40 F		
675	F5 395L		
	40 395L		

LOCATION	ORDER	NOTES PAGE 52
676	L0 391L	
	32 346L	Test for finish.
677	50 5F	
	75 F	
678	40 F	
	S5 F	Special case for (ii), (ij) and
679	50 6F	(jj) elements of A.
	74 F	
680	L4 F	
	40 1F	
681	50 5F	
	75 F	
682	40 F	
	S5 F	
683	50 6F	
	74 F	
684	L4 F	
	40 F	
685	22 370L	
	40 F	
686	L5 1F	
	40 F	
687	L1 1F	
	L4 2F	
688	22 373L	
	40 F	
689	22 374L	
	41 F	
690	22 375L	
	F5 393L	
691	J0 959F	
	50 376L	Write jth row of A back on drum.
692	26 118L	
	00 F	

LOCATION	ORDER	NOTES PAGE 53	
693	L5 33F		
	L0 F		
694	40 33F		
	L5 393L		
695	J0 830F		Write jth row of Q back on drum.
	50 380L		
696	26 77L		
	00 F		
697	F5 393L		
	40 393L		
698	L0 34F		
	32 384L		Test for j = n - 1 ?
699	26 283L		
	F5 392L		
700	J0 894F		
	50 385L		Write ith row of A back on drum.
701	26 118L		
	00 F		
702	22 387L		
	L5 392L		
703	J0 766F		
	50 388L		Write ith row of Q back on drum.
704	26 77L		
	00 F		
705	22 263L		
	00 F		
706	80 F		
	00 F		Constants
707	00 F		
	00 F		
708	00 F		
	00 F		
709	00 F		
	00 F		

LOCATION	ORDER	NOTES	PAGE 54
710	00 F		
	00 F		
711	00 5F		
	00 6F		Constants
712	00 6F		
	00 5F		
713	40 F		
	L3 F		
714	32 400L		
	L5 396L		
715	26 304L		
	L5 45F		
716	40 5F		
	L5 4F		
717	L0 3F		
	40 F		
718	L3 F		
	32 405L		
719	L1 45F		
	40 6F		
720	22 324L		
	L5 45F		
721	40 6F		
	22 324L		
722	L5 41F		
	22 315L		
723	N0 39F		
	50 959F		
724	L0 198L		
	10 20F		
725	42 F		
	L5 F		
726	L4 235L		
	L0 31F		

LOCATION	ORDER	NOTES
727	40 234L	
	26 201L	
	(N12)00K	Input routine.
	00K	
	L3 F	
	34 18F	
	FF 67F	Sum check routine
	24 18F	
	L7 2148F	
	74 2694F	
	26 L	
	26 1N	