UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE R 2 - 105

TITLE — Integral Root $A^{1/p}$ (DOI or SADOI)

TYPE — Closed with one program parameter

NUMBER OF WORDS — 24

TEMPORARY STORAGE — 0-3

ACCURACY — $\pm 2^{-39}$

PARAMETER — If the "Integral Root" subroutine starts at location t, then it is entered (with A in the accumulator) by the following:

s	50 pF
	50 sF
s + 1	26 tF

DURATION — Negligible for the special cases A = 0 $|A| > p \times 2^{-39}$

A table of typical times (in milliseconds) follows:

p/A	.1	.2	.3	.5	.8
2	35	30	35	25	30
3	55	45	40	40	30
4	55	60	50	50	40
10	120	105	120	105	90

For large p and small A the times are considerably greater.

DESCRIPTION — This routine computes the pth root (p, a positive integer, $2 \leq p \leq 1023$) of a 39 binary digit real argument A, $-1 \leq A < 1$. If $|A| > 1 - p \times 2^{-39}$, then $\pm (1-2^{-39})$ is taken as $A^{1/p}$. Another special case is A = 0, in which case $A^{1/p} = 0$. Otherwise $A^{1/p}$ is found by Newton's iteration method, in which

$$x_0 = 1-2^{-39}$$

$$x_{n+1} = x_n + 1/p\ [A/x_n^{p-1}) - x_n]$$

Convergence of x_n to $A^{1/p}$ is assumed when

$$1/p\ [(A/x_n^{p-1}) - x_n] \geq 0.$$

If p is even, of course, A must be non-negative and in this case the non-negative real pth root is found. At the end of the routine the accumulator contains the signed pth root of A.

Rt: 7/21/59

DATE 7/23/53 RT: 9/15/58

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LOCATION	ORDER		NOTES PAGE 1 R 2
	00K (R2)		
0	40 F		N(0) = Argument A
	S5 1F		
1	L4 L		
	42 19L		Set link address
2	00 19F		Set N(21L) = p x 2^{-39}
	42 21L		
3	L3 F		
	32 19L		Exit if A = 0
4	L1 22L		
	40 2F	From 17'	Set $x_0 = 1 - 2^{-39}$
5	40 1F		N(1) = x_n
	L7 F		
6	L4 21L		Exit if $\|A\| \geq 1 - (p-1) \times 2^{-39}$
	L0 22L		
7	36 18L		
	19 37F		
8	40 3F	From 12	
	L0 21L		
9	32 12L		
	50 1F		N(1) = x_n^{p-1}
10	75 2F		
	40 1F		
11	L5 3F		
	L4 23L		
12	26 8L		
	L7 F	From 9	
13	50 22L		
	66 1F		
14	S5 F		Form and test $1/p(A/x_n^{p-1} - x_n)$
	L0 2F		
15	10 39F		
	66 21L		

LOCATION	ORDER	NOTES PAGE 2	
16	S5 F		
	36 18L		
17	L4 2F		$NR_1 = x_{n-1}$
	22 4L		
18	L5 F	From 7,16'	Test sign of A
	36 20L		
19	L3 2F		Exist with $NR_1 = \pm \lvert A \rvert^{1/p}$
	22 F	By 1'	
20	L7 2F	From 20',3' 18'	
	22 19L		
21	00 F		
	00 F	By 2'	$p \times 2^{-39}$
22	80 F		
	00 1F		$-1 + 2^{-39}$
23	00 F		
	00 1F		2^{-39}