UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE V 4 - 178

TITLE	Fourier Analysis
TYPE	Closed
DURATION	$(m^2 + 13m)$ milliseconds
LENGTH	52 words
ACCURACY	3×10^{-12}
TEMPORARY STORAGE	0 through 6 and m consecutive locations specified by preset parameter S6.
ENTRY	When this routine is located at q entry is made by

p	X0 mF
	50 pF
p+1	26 qF

where X = 5 for sine series; X = J for cosine series.

PRESET PARAMETERS

S3	00 F 00 aF	If the function to be analyzed is $f(x)$, $0 < x < L$, values $f_i = f(iL/m)$, $0 \leq i \leq m$, of the function are stored at consecutive locations beginning with a. (There are m + 1 values of the function at equal intervals of length L/m along the interval $0 \leq x \leq L$. The first value is f(0), stored at a. The last value is f(L), stored at m + a.)
S4	00 F 00 bF	This routine uses library routine T 5 - 157 to compute sine or cosine values. b is the location of the first word of routine T 5.
S5	00 F 00 cF	This routine computes values of the coefficients of the sine or cosine expansion of a function. Scaled values of these coefficients are stored by the routine at consecutive locations beginning with c. For sine series the number of coefficients computed is the greatest integer less than or equal to m/2. One more than this is

computed for cosine series.

S6 00 F This routine prepares and uses a table
00 dF of sine or cosine values. This table occupies m locations beginning with d.

NOTES ON USE

This routine computes the first approximately m/2 coefficients of the sine or cosine expansion of a function,

$$f(x) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{L} \qquad (0 < x < L) \qquad (1$$

or

$$f(x) = B_0 + \sum_{n=1}^{\infty} B_n \cos \frac{n\pi x}{L} \qquad (0 < x < L) \qquad (2$$

Values of the function f(x) at equal intervals along the interval from 0 to L are stored prior to entering the routine.

For sine series the coefficients are scaled by 1/2. For cosine series all coefficients except the first, B_0, are scaled by 1/2. B_0 is unscaled.

The accuracy of the computed values of the coefficients depends upon the function being analysed. The sine series represents the function -f(-x), and the cosine series the function f(-x), in the open interval (-L, 0). The sine series represents a function which is zero at the end points, 0 and L, and the cosine represents a function whose first derivative is zero at these points. Both series represent functions which are periodic of period 2L. The accuracy of the computed values of the coefficients depends upon the continuity of this function and the value of m chosen.

More precisely, the error in the integration formula used (the trapezoidal rule) is

$$\Delta A_n = \sum_{i=1}^{\infty} (A_{2im+n} - A_{2im-n}) \qquad (0 < n < m/i)$$

$$\Delta B_0 = \sum_{i=1}^{\infty} B_{2im}$$

$$\Delta B_n = \sum_{i=1}^{\infty} (B_{2im+n} + B_{2im-n}) \qquad (0 < n < m/2)$$

Thus the error tends to be large if the higher (relative to m) harmonics are not negligible. For example, the relative error in A_1 for a function whose sine coefficients fall off as $1/n$ is about 0.5% with m = 32; whereas if the sine coefficients fall off as $1/n^3$ the relative error in A_1 is about 0.0015% for m = 32. It will also be noticed from the error formula that if, for instance, the even harmonics of the function are zero, the error due to the integration formula for their even coefficients will also be zero. The only error for these coefficients in this case will be the error due to the routine itself, which has a maximum value of 3×10^{-12}.

A complete Fourier series representation of a function,

$$f(x) = B_0 + \sum_{n=1}^{\infty} [A_n \sin \frac{n\pi x}{L} + B_n \cos \frac{n\pi x}{L}] \quad (-L < x < L)$$

may be obtained using this routine. The sine coefficients are obtained by finding the sine coefficients of the function

$$f_1(x) = 1/2\ [f(x) - f(-x)] \qquad (0 < x < L)$$

The cosine coefficients are obtained by finding the cosine coefficients of the function

$$f_2(x) = 1/2\ [f(x) + f(-x)] \qquad (0 < x < L)$$

The series thus obtained will represent the function over the interval $(-L < x < L)$ and will be periodic of period 2L. However, to obtain this series, values of the function must be known at an odd number $(2m + 1)$ of points. m itself may be either even or odd.

This routine leaves with the address, specified by preset parameter S5, of the first coefficient computed as the right hand address of the word in the accumulator.

DATE	April 4, 1955
CODED BY	C.C. Farrington
APPROVED BY	J.P. Nash

LOCATION	ORDER		NOTES PAGE 1
0	41 3F		
	41 4F		
1	41 5F		
	L5 49L		
2	40 10L		
	46 5F		Set n = 1
3	K5 F		
	42 47L		
4	46 4F		
	10 20F		
5	42 3F		
	36 8L		Sine or cosine?
6	19 3F		
	L4 10L		
7	40 10L		For cosine values set n = 0
	41 5F		
8	41 6F	from 5	
	L5 6F	from 15	Prepare table
9	J0 6F		
	66 3F		
10	S(5) 1F	by 2, 7	
	50 10L		
11	26 S4		
	40 ( )S6	by 13, 16	
12	F5 6F		
	40 6F		
13	F5 11L		
	42 11L		
14	L5 3F		
	F0 6F		
15	32 8L		
	L5 51L		
16	42 11L		
	L4 4F		
17	46 37L		
	L5 5F	from 46	Compute coefficient

LOCATION	ORDER		NOTES PAGE 2
18	F4 20L		
	40 24L		
19	F5 50L		
	40 6F		
20	50 S6		Integration
	7J S3		
21	40 2F		
	50 2F		
22	75 50L		
	40 1F		
23	S5 F	from 35	
	40 F		
24	50 ( )S6	by 32	
	7J ( )S3]	by 18, 29	
25	40 2F		
	50 2F		
26	L5 F		
	74 6F		
27	L4 1F		
	40 1F		
28	41 2F		
	L5 24L		
29	F4 5F		
	40 24L		
30	L0 48L		
	42 2F		
31	L0 4F		
	36 34L		
32	L4 48L		
	46 24L		
33	L1 6F		
	40 6F		
34	L5 3F		
	F0 2F	from 31	
35	36 23L		
	S5 S5		

LOCATION	ORDER	NOTES	
36	40 F		
	50 S6		
37	75 ( )S3	by 17	
	10 1F		
38	40 2F		
	50 2F		
39	L5 F		
	74 6F		
40	L4 1F		
	66 3F		
41	S5 F		
	40 ( )S5	by 42, 47	
42	F5 41L		Prepare to compute next coefficient
	42 41L		
43	L5 5F		
	L4 49L		
44	46 5F		
	L5 4F		
45	10 1F		
	L0 5F		
46	32 17L		
	L5 35L		
47	42 41L		
	22 ( )F	by 3	Exit
48	J0 S6		Test constant
	7J S3		
49	S5 1F		Order used for sine series only
	50 10L		
50	00 F		Scaling constant
	00 1F		
51	00 S3		
	00 S6		