TITLE:

TYPE:

NUMBER OF WORDS:

ACCURACY:

Floating Point Simpson's Rule Integration (SADOI Only)

Closed

41

The truncation error in Simpson's Rule in taking 2
steps each of length h is:

$$\frac{h^5 f^4(\xi)}{90}$$

where ξ is some point in the interval.

DURATION:

$[(32s+ A) + 73 + A]$ m.s. where s is the number of
steps and A is the time in m.s. required by the
auxiliary.

DESCRIPTION:

An approximate value for the integral
$$I = \int_a^b f(x)\, dx$$
is obtained by

$$I = \frac{h}{3}\left\{ f(x_0) + f(x_n) + 4[f(x_1) + f(x_3) + \ldots \right.$$
$$+ f(x_{n-1})] + 2[f(x_2) + f(x_4) + \ldots$$
$$\left. + f(x_{n-2})]\right\}$$

where $h = x_i - x_{i-1}$. There must be an even number of
intervals, n, given by an odd number, n + 1, of points
$x_0, x_1, \ldots, x_n$. The **minimum** number of points is 3, for
which $I = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)]$.

The size of the interval h can be changed at any
point with n even; thus, for example we may have for
two intervals

$$I = \frac{h_1}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3)$$
$$+ f(x_4)] + \frac{h_2}{3}[f(x_4) + 4f(x_5) + 2f(x_6)$$
$$+ 4f(x_7) + \ldots + f(x_n)].$$

ENTRY:

Entry is made from <u>fixed</u> <u>point</u> in the **standard** way with control transferred to symbolic address (SIMP), i.e.

p	50 p
p + 1	26 (SIMP)

Upon entry the following parameters must be stored in the following order at symbolic location (L):

$$n_1, h_1, x_{01}, n_2, h_2, x_{02}, \ldots, n_k, h_k, x_{0k}, 0$$

with the zero following the last group to denote the termination, where n_i (a positive, even, fixed point integer, ie. $n_i \times 2^{-39}$) = number of steps to be taken with this step length, h_i (a floating point number) = length of step, x_{0i} (a floating point number) = starting point of integration with this length, $i = 1, 2, \ldots, k$.

Hence the step length can be changed without leaving the routine.

EXIT:

Control is returned, in fixed point, to the right hand order in p + 1 with the value of the integral in the floating accumulator.

AUXILIARY:

The auxiliary subroutine for computing (or otherwise determining), $f(x)$ must be located at symbolic location (AUX). It is entered from EA 1 in fixed point using an 8J order, with x in the floating accumulator. It should begin as follows:

(AUX)	L5 2S4	
	40 (SAVE)	
	32 1(AUX)	Waste
	50 1(AUX)	Enter A1 with x in the
	26 S4	floating accumulator

The auxiliary should place the value of $f(x)$ in the floating accumulator and exit should be made as follows:

L5 (SAVE)
40 2S4
26 29S4

ASSEMBLY:

The floating accumulator must be in the locations specified by S3 and LS3. Library routine A 1 must begin at the location specified by S 4. If the user desires other of the floating point subroutines he will find it most expedient to use routine A 6 by calling it from the drum.

Routine EA 1 uses b-boxes 0 and 1 of A 1 and the following symbolic addresses:

(1), (J), (K), (N), (S) and (SIMP).

DATE March 5, 1959

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lgr

LOCATION	ORDER		NOTES
	00 K		
0 (SIMP)	K5 F		
	42 5L		
1	41 (S)		Clear grand sum
	L5 12L		
2	42 4L		
	L4 33L		
3	42 7L		
	L4 33L		
4	42 8L		
	L1 (L)	by 2L	$A = -n_i$
5	40 F		
	32 F	link	$+ \to$ done
6	L1 F		$A = n_i$
	10 1F	$n_i/2$	
7	42 15L		
	L5 1(L)	by 3L	$A = h_i$
8	40 (K)		
	L5 2(L)	by 4L	$A = x_{0i}$
9	40 (N)		Set x_0
	41 (J)		
10	41 1(J)		
	22 14L		
11	L5 4L	27 L	
	L4 32L		
12	26 2L		
	00 (L)		
13	00 F		
	00 F		
14	00 F		
	50 14L		Enter F.P.
15	26 S4		
	1K [$n_i/2$]F	by 7L	Set loop
16	85 (N)		$F = x_0$
	8J (AUX)		Get $f(x_0)$

LOCATION	ORDER	NOTES
17	8S (1)	
	OK 2F ← 31 L	Set binary switch at 20L
18	85 (N) ← 29 L	
	84 (K)	$x \rightarrow x + h$
19	8S (N)	
	8J (AUX)	
20	03 28L	Transfer every other time
	13 30L	Transfer $n_i/2 - 1$ times
21	84 (1)	
	8S (1)	
22	8K 4F	
	87 (J)	
23	84 (1)	
	8S (1)	
24	8K 2F	
	87 1(J)	
25	84 (1)	
	86 34L	$\div 3$
26	87 (K)	Multiply by h_i
	84 (S)	Add to grand sum
27	8S (S)	
	8J 11L	
28	84 (J) ← from 20L	
	8S (J)	
29	8K F	Set F = 0
	83 18L	
30	84 1(J) ← from 20L	
	8S 1(J)	
31	8K F	Set F = 0
	82 17L	
32	00 F	
	00 3F	
33	00 F	
	00 1F	
34	26 1638F	= 3 in F.P.
	66 1729F	

LOCATION	ORDER		NOTES
35	(1)	OO F	
		OO F	
36	(J)	OO F	
		OO F	
37		OO F	
		OO F	
38	(K)	OO F	
		OO F	
39	(N)	OO F	
		OO F	
40	(S)	OO F	
		OO F	