UNIVERSITY OF ILLINOIS
DIGITAL COMPUTER

ILLINOIS CODE M 10 - 153

TITLE

Estimation of Communalities by the Method of Maximum Likelihood.

TYPE

Entire program.

METHOD OF USE

The program is read into the memory in the usual way. After a short time, Illiac will stop. Read in parameters and the elements of the correlation matrix. The computer will stop and is now prepared to read in the estimated elements of the matrix G. Replace the original tape in the reader and restart the computer. The black switch must be in the ignore position. If convergence has not occurred in the allotted time, raise the black switch to the obey position. After a short time the computer will come to a stop. At that time raise the white switch up and down. This will read in the print routine and the results will be printed out shortly.

PUNCHING OF TAPES

At the beginning of each data tape four parameters are necessary. They are as follows:

(1) Let s = size of sample. Then place sS on the tape.

(2) Let f = the number of estimated factors. Place fF on the tape.

(3) Let n = the number of variables. Place nN on the tape.

(4) Let j = the number of places to be printed. Place jJ on the tape.

Since the chi-square value varies with the number of factors, it is necessary to provide several values. These can be found in Table I for the 5 % level.

Each value (which must be scaled by 10^{-4}) is punched as a sign followed by up to 12 decimal digits. The final value in the sequence is terminated by an N. The values of the correlation matrix scaled by 10^{-2} are next punched on the tape. The lower off-diagonal elements and diagonal elements are punched, row by row, as a sign followed by up to 12 decimal digits. The final fraction in the sequence is terminated by an N. It is necessary to make some estimation of the matrix G. The values scaled by 10^{-1} should be punched as a sign followed by up to 12 decimal digits. The last value is followed by an N.

MATHEMATICAL METHOD

The essential steps in the iterative process of estimation and tests of significance may be summarized as follows:

(1) Start with a trial diagonal matrix G and obtain the solution to the determinantal equation $|GRG - \lambda I| = 0$. This gives the first approximation to the diagonal elements of G.

$$g_i = \sqrt{1 + \ell_{i1}^2 (\lambda_1/\lambda_e - 1) + \ell_{i2}^2 (\lambda_2/\lambda_e - 1) + \ldots + \ell_{if}^2 (\lambda_f/\lambda_e - 1)}$$

where $\underset{\sim}{\ell}_1, \ldots, \underset{\sim}{\ell}_f$ are the f(number of factors assumed) latent normalized vectors corresponding to the f largest roots $\lambda_1, \ldots, \lambda_f$ and

$$\lambda_e = (\lambda_{f+1} + \lambda_{f+2} + \ldots + \lambda_n)/\ n\text{-}f$$

(2) This process is repeated till g_i converges to about four decimal places. When g_i converges the sum

$$\Sigma\ g_i^2 - (\lambda_1 + \ldots + \lambda_f) = \lambda_{f+1} + \ldots + \lambda_n \rightarrow (n\text{-}f)$$

(3) The estimated factor loading's at any stage are

$$\sqrt{\lambda_1 - 1}\, \underset{\sim}{\ell}_1 G^{-1}, \sqrt{\lambda_2 - 1}\, \underset{\sim}{\ell}_2 G^{-1}, \ldots, \sqrt{\lambda_f - 1}\, \underset{\sim}{\ell}_f G^{-1}$$

The estimated communalities are, at any stage,

$$h_i^2 = (g_i^2 - 1)/g_i^2$$

(4) The test for a specified number k of factors is'

$$\Lambda_f = -\left(S - 1 - (2h + 5)/6 - (2f)/3\right)\left\{\ell n \lambda_{f+1} + \ldots + \ell n \lambda_n - (n-f)\ell n\, \lambda_e\right\}$$

which is a chi-squared test on $[(n-f)-n-f]/2$ degrees of freedom. This test is made when Λ_f converges to about five places. If Λ_f is greater than the chi-square value that the number of factors in increased by one.

ESTIMATION OF G

Because the convergence of the iterative process is somewhat slow, it is desirable to choose values of G which will minimize the number of iterations. It has been shown* that the squared multiple correlations are the minimum estimates of sommunalities. Consequently, the square root of diagonal elements of the inverse of the correlation matrix form a minimum estimates for the matrix G.

THE PRINT-OUT

After the first iteration, the latent roots are printed out. The Λ_f values are printed out after each iteration. This is followed by the elements of the matrix G, the communalities, the largest latent roots, and the factor loadings. If too many factors are being extracted or if there are too few values of chi-square, then ten F's will be punched out and the computer will stop.

*L. Guttman, Multiple Rectilinear Prediction and the Resolution into Components, Psychometrika, Vol. 5 (1940) pp. 75-90

TABLE I

DEGREES OF FREEDOM

f/n	5	6	7	8	9	10	11	12	13	14	15	16	17
1	5	9	14	20	27	35	44	54	65	77	90	104	119
2	1	4	8	13	19	26	34	43	53	64	76	89	103
3			3	7	12	18	25	33	42	52	63	75	88
4				2	6	11	17	24	32	41	51	62	74
5					1	5	10	16	23	31	40	50	61
6							4	9	15	22	30	39	49
7								3	8	14	21	29	38
8									2	7	13	20	28
9										1	6	12	19
10												5	11
11													4

CHI-SQUARE* AT 5% LEVEL

1	11.1	16.9	23.7	31.4	40.1	49.8	60.2	71.9	84.5	98.2	112.9	128.5	145.2
2	3.84	9.49	15.5	22.4	30.1	38.9	48.6	59.0	70.7	83.4	97.1	111.7	127.4
3			7.81	14.1	21.0	28.9	37.7	47.4	57.8	69.5	82.2	95.9	110.6
4				5.99	12.6	19.7	27.6	36.4	46.2	56.6	68.4	81.1	94.8
5					3.84	11.1	18.3	26.3	35.2	45.0	55.5	67.2	79.9
6							9.49	16.9	25.0	33.9	43.8	54.3	66.1
7								7.81	15.5	23.7	32.7	42.6	53.1
8									5.99	14.1	22.4	31.4	41.3
9										3.84	12.6	21.0	30.1
10												11.1	19.7
11													9.49

*When degrees of freedom are greater than 30 an approximation of chi-square is made.

LOCATION	ORDER	NOTES	PAGE 1
	CODE X1		Decimal Order Input
	00 80K		
	CODE N2		Input a Sequence of Decimal Fractions
	00 20K		
0	41 5F		
	41 F	From 8	
1	81 4F	From 5	
	L0 37L		
2	32 5L		
	L4 37L		Read in parameters and convert
3	50 F		
	74 37L		
4	S5 F		
	40 F		
5	26 1L		
	42 6L	From 2	
6	L5 F		
	40 ( )F	By 5	
7	F5 5F		
	40 5F		
8	L0 39L		
	32 L		
9	L5 2F		
	00 1F		
10	40 F		
	L5 4F		
11	00 2F		
	L4 F		
12	F4 37L		
	10 39F		
13	66 38L		
	S1 F		
14	L4 1F		
	40 9F		Compute $\left\{ S - 1 - \frac{2n+5}{6} - \frac{2f}{3} \right\}$
15	L5 2F		

LOCATION	ORDER		NOTES PAGE 2
	50 2F		
16	74 2F		
	S5 F		
17	10 1F		
	40 11F		Compute $\frac{n(n+1)}{2}$
18	L4 36L		
	40 7F		$331 + \frac{n(n+1)}{2}$
19	50 2F		
	74 2F		
20	S5 F		
	40 6F		$n^2 + n$
21	00 20F		
	46 30L		
22	L5 6F		
	L4 11F		
23	40 8F		$3/2\ (n^2 + n)$
	00 20F		
24	46 32L		
	L5 2F		
25	L0 4F		
	40 10F		n-f
26	L5 2F		
	40 4F		
27	L4 2F		
	40 5F		2n
28	50 988F		
	50 28L		Read in chi-square values
29	26 80F		
	22 30L		
30	50 ( )F	By 21	
	50 30L		Read in correlation matrix R
31	26 80F		
	24 32L		Stop, prepare to read in Matrix G
32	50 ( )F	By 24	
	50 32L		

LOCATION	ORDER	NOTES PAGE 3
33	26 80F	
	50 40L	n-f/100
34	75 10F	
	S5 F	
35	40 12F	
	24 999F	
36	00 F	
	00 331F	
37	00 F	
	00 10F	
38	00 F	
	00 6F	
39	80 F	
	00 4F	
40	00 F 00 0100	
	0000 0000 J	
	24 20N	
	00 959K	
	Code P1	Print one Number Fractional or Integer in a Manner determined by a Program parameter
	00 20K	
	Code M0	Eigenvalue - Eigenvector Program
	00 152K	
	Code R1	Square Root Routine
	00 286K	
	Code S1	Natural Logarithm
	00 945K	
0	92 131F	
	L5 13F	Print out chi-square criterion
1	52 74F	
	50 1L	
2	26 959F	
	L5 191F	
3	22 217F	

LOCATION	ORDER	NOTES	PAGE 4
	00 F		
	00 950K		
0	41 4F	From 169F	
	L3 127F		
1	36 170F		Is this first iteration?
	41 127F		If so, print out all latent roots.
2	41 5F		
	92 147F		
3	92 131F	From 8	
	L5 (331)F	By 6	
4	52 72F		
	50 4L		
5	26 9L		
	F5 3L		
6	42 3L		
	F5 5F		
7	40 5F		
	L0 278F		
8	36 3L		
	26 170F		
	00 161K		
0	L5 147F	From 115	
	42 3L		
1	46 3L		
	42 107L		
2	41 F		
	50 F		
3	L5 ( )F	By 1,6	
	40 ( )F	From 8 By 0, 6	Store latent roots in consecutive
4	F5 F		locations.
	40 F		
5	K4 F		
	00 20F		
6	F4 3L		

LOCATION	ORDER	NOTES	
	40 3L		
7	L5 F		
	L0 117L		
8	36 3L		
	26 950F		
9	41 5F	From 958F	
	41 6F		
10	49 8F		
	41 7F	From 30	
11	L5 147F		
	42 12L		
12	46 13L		
	L7 ( )F	By 11, 16	From 18
13	L2 ( )F	By 12, 15	Find smallest latent roots.
	32 15L		
14	L5 12L		
	00 20F		
15	46 13L		
	F5 12L	From 13	
16	42 12L		
	F5 7F		
17	40 7F		
	L0 117L		
18	32 12L		
	L5 13L		
19	46 21L		
	46 25L		
20	46 15F		
	41 9F		
21	L5 ( )F	From 19	
	40 2F		
22	40 16F		
	L4 5F		
23	40 5F		Sum smallest latent roots
	50 23L		

LOCATION	ORDER	NOTES	PAGE 6
24	22 286F		Find logarithm of smallest latent
	L5 142F		roots.
25	40 ( )F	From 19	Store 1 - 2^{-39} in locations of
	41 10F		smallest latent roots.
26	50 F		
	7J 118L		
27	L4 120L		
	L4 6F		
28	40 6F		Sum logarithms of smallest latent
	F5 4F		roots.
29	40 4F		
	L0 119L		
30	32 10L		
	50 S8		Constant
31	L5 5F		
	50 31L		
32	26 286F		
	40 7F		
33	L5 116L		
	50 33L		
34	26 286F		
	L0 7F		
35	40 F		
	50 F		
36	7J 118L		
	40 F		Compute chi-square criterion
37	50 119L		
	J0 142F		
38	75 F		
	00 39F		
39	L4 6F		
	40 F		
40	50 F		
	71 123L		
41	L1 13F		

LOCATION	ORDER	NOTES	PAGE 7
	S4 F		
42	40 F		
	S5 F		
43	40 13F		
	19 15F		
44	L2 F		
	32 45L		Has chi-square value converged?
45	26 56L		If not, transfer control to 56
	L3 (988)F	From 44	By 52
46	36 124L		Are too many factors being extracted?
	40 F		If so, transfer to 124.
47	L7 F		
	L0 13F		
48	36 56L		Does criterion exceed chi-square
	L5 119L		value? If so, increase number of
49	L0 145F		factors by one. Otherwise, transfer
	42 119L		control to 56.
50	L5 116L		
	L0 121L		
51	40 116L		
	F5 45L		
52	42 45L		
	L5 15F		
53	46 54L		
	L5 16F		
54	40 ( )F	By 53	
	L5 5F		
55	L0 16F		
	40 5F		
56	26 945F	From 45, 48	
	42 77L	From 948F	
57	42 81L		
	L5 68L		
58	22 59L		
	L5 3F	From 85	

LOCATION	ORDER	NOTES
59	L4 117L	
	42 3F	
60	41 4F	
	L5 147F	
61	42 62L	
	L5 121L	
62	40 1F	
	F7 ( )F	By 61, From 76
63	36 64L	
	22 73L	
64	L5 62L	From 63
	42 66L	
65	L4 3F	
	42 69L	
66	42 71L	
	50 F	
67	75 116L	
	66 5F	
68	L1 121L	
	S4 SS	Estimate g_i^2
69	40 2F	
	50 ( )F	By 65
70	7J 2F	
	40 2F	
71	41 F	
	50 ( )F	By 66
72	7J 2F	
	L4 1F	
73	40 1F	
	F5 62L	From 63
74	42 62L	
	F5 4F	
75	40 4F	
	L0 117L	
76	32 62L	

LOCATION	ORDER	NOTES	PAGE 9
	50 76L		
77	22 152F		
	L0 ( )F	By 56, 82	
78	40 1F		Test to see if g_i has converged.
	19 13F		
79	L2 1F		
	36 81L		
80	41 8F		
	36 81L		
81	L5 2F	From 79	
	40 ( )F	By 57, 83	
82	F5 77L		
	42 77L		
83	42 81L		
	F5 9F		
84	40 9F		
	L0 117L		
85	32 58L		
	L3 8F		
86	34 87L		
	26 999F		Have all the g_i's converged?
87	L5 151F		If so, bring in print routine.
	42 89L	From 97	
88	F5 89L		
	42 92L		
89	F5 142F		
	40 ( )F	From 87	
90	F4 8F		
	42 8F		
91	L0 117L		
	32 97L		
92	41 1F		
	41 ( )F	From 88, 95	By 93
93	F5 92L		
	42 92L		

LOCATION	ORDER	NOTES	
94	F5 1F		
	40 1F		Generate identity matrix
95	L0 117L		
	32 92L		
96	L5 89L		
	F4 117L		
97	22 87L		
	L5 122L	From 91	
98	42 104L		
	46 104L		
99	F5 142F		
	40 2F		
100	41 3F	From 114	
	F5 2F		
101	40 2F		
	19 18F		
102	L4 104L		
	46 104L		
103	L5 30L		
	42 105L		
104	50 ( )F	By 98, 102	From 112
	7J ( )F	By 98, 108	
105	40 F		
	50 ( )F	By 103, 109	
106	75 F		Compute GRG
	66 121L		
107	S5 F		
	40 (331)F	By 1, 110	
108	F5 104L		
	42 104L		
109	F5 105L		
	42 105L		
110	F5 107L		
	42 107L		
111	F5 3F		

LOCATION	ORDER	NOTES	PAGE 11
	40 3F		
112	L0 2F		
	36 104L		
113	L4 2F		
	L0 117L		
114	36 100L		
	50 114L		
115	26 20F		Compute eigenvalues and eigenvectors
	26 L		of GRG
116	00 F		
	00 SN	By 51	
117	80 F		
	00 S4		
118	00F 00 0032		
	0000 0000 J		
119	80 F		
	00 (SK)	By 49	
120	00F 00 0004		
	6051 7019 J		
121	00F 00 0100		
	0000 0000 J		
122	00 1023S8		
	00 S6		
123	00 F		
	00 S9		
124	92 934F		Punch out 10 F's and stop, if too
	0F F		many factors are being extracted.
	00 15K		
	41 8F		
	26 248F		
	26 15N		
	00 10K		
0	09 10F		
	L5 128F		

LOCATION	ORDER		NOTES
1	00 1F		
	40 1F		
2	00 2F		
	L4 1F		
3	40 3F		Printing interlude;
	L4 128F		Compute parameters necessary for
4	40 4F		Code P1
	50 191F		
5	J0 7L		
	S5 F		
6	40 8F		
	26 999F		
7	00 F		
	00 1023F		
	26 10N		
	00 20K		
0	41 7F		
	92 983F		
1	92 143F		
	41 8F		
2	L5 (S8)	By 13,	From 16
	40 9F		
3	52 1S3		
	50 3L		Print out G
4	26 959F		
	50 9F		
5	75 9F		
	40 F		
6	L0 282F		
	66 F		
7	S5 F		
	40 (80)F	By 14	Compute communalities and store.
8	92 131F		
	L5 57L		
9	50 8F		

LOCATION	ORDER	NOTES	
	66 9F		
10	41 10F		
	S5 F		
11	40 (S8)	By 12	Compute G^{-1}
	19 18F		
12	L4 11L		
	46 11L		
13	46 2L		
	F5 7L		
14	42 7L		
	F5 7F		
15	40 7F		
	L0 278F		
16	36 2L		
	92 770F		
17	92 143F	From 22	
	L5 (80)F	By 20	
18	52 S4		
	50 18L		Print out communalities
19	26 959F		
	92 131F		
20	F5 17L		
	42 17L		
21	F5 8F		
	40 8F		
22	L0 278F		
	32 17L		
23	92 143F		
	F7 (331)F	By 33,	From 35
24	36 25L		
	26 33L		
25	L5 23L	From 24	
	42 26L		
26	42 32L		
	L5 ( )F	By 25	

LOCATION	ORDER	NOTES
27	40 4F	
	36 28L	
28	52 2S3	
	50 28L	Print out largest latent roots
29	26 959F	
	41 F	
30	92 131F	
	L5 4F	
31	L0 282F	
	50 31L	
32	26 152F	
	40 ( )F	By 26 Compute $\sqrt{\lambda_i - 1}$ and store
33	F5 23L	From 24
	42 23L	
34	F5 10F	
	40 10F	
35	L0 278F	
	32 23L	
36	41 8F	
	92 143F	
37	50 S8	Constant
	L5 37L	From 56
38	46 43L	
	F7 (331)F	By 54
39	36 40L	
	22 53L	
40	L5 38L	From 39
	42 42L	
41	L4 229F	
	42 44L	
42	41 9F	
	50 ( )F	From 40, 52
43	7J ( )F	By 38, 48
	40 F	
44	50 F	

LOCATION	ORDER	NOTES
	75 ( )F	From 41,50 Compute factor loadings: $\sqrt{\lambda_1 - 1}\, l_{v1}\, G^{-1}$
45	66 57L	
	S1 F	
46	52 S4	
	50 46L	Print out factor loadings
47	26 959F	
	19 18F	
48	L4 43L	
	40 43L	
49	L5 278F	
	L4 44L	
50	42 44L	
	92 131F	
51	F5 9F	
	40 9F	
52	L0 278F	
	32 42L	
53	92 135F	
	F5 38L	From 39
54	42 38L	
	F5 8F	
55	40 8F	
	L0 278F	
56	32 37L	
	OF F	
57	OOF OO 1000	
	0000 0000 J	
	26 20N	

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