binary digits 1 in the positions corresponding to terms which are present in an expansion and by binary digits 0 in positions corresponding to terms which are absent. For example, the expansion $x^3\ x^2\ \bar{x}^1 + \bar{x}^2 + \bar{x}^3\ x^1$ would be represented by 1's in the positions 1, 23, and 15 and by zeros elsewhere.

In a set of memory locations containing an expansion the digit positions 2^{-13} through 2^{-39} of the first memory location hold the first 27 terms of the expansion (i.e. terms 0 through 26), while the next memory location holds the next 27 terms in corresponding positions, and so on. Hence we see that if 7 input variables are used, our expansions will be represented by $3^7 = 2187$ binary digits and each logical register will occupy $3^4 = 81$ memory locations.

In an expansion of the type discussed here one may regard the operations between terms to be either or or exclusive or. Simplification procedures of (for example) the type $\bar{x}^5\ x^3\ x^1 \vee \bar{x}^5\ \bar{x}^3\ x^1 = \bar{x}^5\ x^1$ and $\bar{x}^5\ \bar{x}^3\ x^1 + \bar{x}^5\ x^1 = \bar{x}^5\ x^3\ x^1$ may then be carries out by a short set of interpretive orders. Other simplification processes, such as $x^2\ x^1 \vee \bar{x}^3\ x^2 \vee x^1\ x^2 = x^3\ x^1 \vee \bar{x}^3\ x^2$ are then carried out by other sets of interpretive orders.

DESCRIPTION OF INTERPRETIVE ORDERS

Let F be the expansion in the logical accumulator. Eight types of algebraic operations may be carried out by interpretive orders.

0L	n	Replace F by F(n),
1L	n	Replace F by $\overline{F(n)}$. ($\overline{F(n)}$ is the expansion having terms in just those positions where F(n) has no terms)
2L	n	Replace F by F x F(n). (F x F(n) is the expansion having terms where both F and F(n) have terms)
3L	n	Replace F by F x $\overline{F(n)}$.
4L	n	Replace F by F v F(n). (F v F(n) is the expansion having terms where F and/or F(n) have terms)
5L	n	Replace F by F v $\overline{F(n)}$
6L	n	Replace F by F + F(n). (F + F(n) is the expansion having terms where either F or F(n) but not both, has a term).
7L	n	Replace F by F + $\overline{F(n)}$.

Nine types of shift operations may be carried out by means of interpretive orders. These orders are combined with the type of algebraic operation defined by A. These orders are of the form 4AB n where A may be 0,1,...,7 and B is of the form $4M_1 + M_2$, with $0 \leq M_1, M_2 < 3$.

When $M_2 = 0$ we extract from F(n) all terms containing x^i as a factor. The selection of i is described under the SO, LO orders.

When $M_2 = 1$ we extract from F(n) all terms containing $\bar{x}^i$ as a factor.

When $M_2 = 2$ we extract from F(n) all terms not containing x^i or $\bar{x}^i$ as a factor.

When $M_1 = 0$ we shift the extracted terms to the positions of the corresponding terms having x^i as a factor and then perform the algebraic operation defined by A.

When $M_1 = 1$ we shift to the $\bar{x}^i$ positions before performing A.

When $M_1 = 2$ we shift to the positions corresponding to either x^i or $\bar{x}^i$ before performing A.

Hence we see that the order 49 n would take those terms of the form $\bar{x}^i$ a from F(n), shift them so they become terms of the type a and place them in the accumulator without clearing F.

Seven "red tape" orders are provided.

80 n — Replace F(n) by F

90 n (JO n) — Leave the interpretive routine and obey the machine order on the right, (left) side on n.

KO n (FO n) — Transfer control to the interpretive order on the right, (left) side of n if the logical accumulator is not empty. Otherwise ignore the order.

SO n — Transfer control to the interpretive order on the right (left) side of n. This order is obeyed cyclically p-1 times (where p is the number of input variables) and disobeyed on the pth time the order is inspected. After such an order has been inspected the input variable index i for shift orders will be stepped by one when the next shift order is inspected and before it is obeyed. When an SO or LO order is disobeyed the input variable index i is cleared to zero as at the beginning of the program and the next shift order will step it to one before it is obeyed. The SO or LO order may be used in this way to carry

out the same simplification process for each of the p variables in turn or, alternatively, it may be used to do the same operation p times.

One caution applies to the use of shift orders. It is not possible to perform a shift on the accumulator itself if $M_2 < M_1$. For example, one could construct the canonical expansion of F without using any of the outside memory by the series of orders

r	62	S4
	66	S4
r+1	6K	S4
	LO	rL

but one could not construct the corresponding net expansion by

r	61	S4
	69	S4
	65	S4
	LO	rT.

because one cannot perform the operation 69 with the accumulator upon itself.

USE OF THE PARAMETER Q:

The 15th word in the subroutine (designated by 14S5) is left empty so that the parameter Q may be inserted if desired. When $Q \times 2^{-39}$ is inserted, then the ith ternary digit in Q is added, modulo 3, to M_2 whenever a shift order is obeyed. (i is the index of the variable being altered in the shift). In this way it is possible to alter the effect of the shift orders for different variables. A primary use for this parameter is to permit the formation of different normal expansions, or to isolate specific terms in a given expansion.

APPENDED SUBROUTINES:

The following appended subroutines will be designated by the orders which cause them to be entered.

JO 179S5 This subroutine causes the expansion in the logical accumulator to be printed without damage to the expansion. Terms like $x^5 \bar{x}^3 \bar{x}^1$ would be printed as 0 2 1 2 1 if 5 input variables were used. After each term a space in printed, and after each 8 terms the carriage is returned. The expansion is terminated by the letter N. Before each print, two carriage returns are always provided. To avoid these use JO 180S5.

JO 155S5 This subroutine causes an expansion to be read into the logical accumulator in the form of a truth table. If p variables are used, one must read in 2^{p-2} sexadecimal characters, the binary digits of which correspond to the truth values of the function. If $p < 3$ one must provide 2 sexadecimal characters.

JO 211S5 This subroutine reads an expansion into the logical accumulator when it is in the form written by JO 179S5. That is, the individual terms must be typed in the ternary representation and the expansion terminated by the symbol N.

To reenter the code after having left it by means of a 90 or JO order one can go to the right hand order of 123 S5. When this is done the interpretive orders following the 90 or JO orders will be selected and obeyed in turn. To end the routine one can go to an OF order by means of 90 102S5.

EXAMPLE OF THE USE OF THE SUBROUTINE

A simple code which carries out the Harvard minimizing chart method serves as an example of the use of the interpretive orders.

0	JO 155S5
	80 600F

These orders read an expansion into the accumulator and store it at 600.

1	3L S4
	80 700F

3LS4 clears the accumulator since none of the terms in the expansion F agree with $\bar{F}$. 80 700F then clears 700.

2	48 600F
	29 600F

This pair of orders carries out the transformation $x^i \; a \; v \; \bar{x}^i \; a = a$.
The first time they are obeyed i = 1.

3	42 S4
	46 S4

These orders carry out $a \longrightarrow a \; v \; x^i \; a \; v \; \bar{x}^i \; a$.

4	4L 700F
	SO 1L

These orders cause the quantity in the accumulator to be "or"ed into 700F and the process to be repeated for each index i. After completion of this process we will have all possible terms if p-1 factors which can be generated from the expansion of 600. In addition to these terms, we have in the accumulator all terms which were used to generate them.

5	6L 600F
	S0 L

6L 600F followed by 80 600F causes the new terms having p - 1 factors to be inserted in 600F and the unneeded terms eliminated. S0 L causes the process to be repeated p times so that terms having the fewest possible factors will be generated.

6	J0 179S5
	90 102S5

J0 179S5 causes the resulting simplified expansion to be printed and 90 102S5 stops the machine.

RT: 3/17/60

DATE December 15, 1953

PROGRAMMED BY D. E. Muller

APPROVED BY J. P. Nash

ns

LOCATION	ORDER	NOTES PAGE 1 Q 1
	00K(Q1)	
	26 1000N	
0	00 59F	Enter
	L4 8L	
1	L0 9L	
	40 2L	
2	50 F	Select order
	S5 20F	
3	46 24L	Store address
	36 26L	Go to algebra
4	00 1F	Right or left
	40 F	
5	50 F	Choose instruction
	01 22F	
6	L4 89L	
	46 7L	
7	26 F	Constants of entry
	00 19L	
8	00 1F	
	S5 20F	
9	5S F	
	S5 F	
10	5S 1F	
	S5 F	
11	00 F	
	00 1F	1
12	00 F	
	00 3F	3
13	00 F	
	00 27F	27
14	00 F	
	00 F	Q
15	00 F	
	00 F	q
16	00 F	
	00 F	C
17	00 F	
	00 F	3^{k-1}

UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

AUXILIARY
LIBRARY ROUTINE Q 1 - 81

TITLE: Logical Algebra Subroutine (SADOI Only)

TYPE: Interpretive routine with 87 orders and 3 appended subroutines. It is entered as a closed routine and left by a 90 or J0 order.

NUMBER OF WORDS: 230

TEMPORARY STORAGE: 0, 1, 2, 3, 4

PARAMETERS:

(1) p, the number of independent variables, may either be inserted in address S3 before reading the program or read in later by using a subroutine.

(2) Location of the first word of the logical accumulator is addressed as S4.

(3) For convenience the routine itself may be addressed as S5.

DESCRIPTION: This routine is intended to aid in the design of switching circuits. It permits the manipulation of expansions in Boolean algebra by means of intepretive "orders" which it carries out, one order at a time. In this way, systematic simplification procedures, such as the Harvard minimizing chart procedure, and the simplified polynomial procedure which is used here, may be prepared as programs for the machine.

Polynomial expansions are handled by the subroutine in which the terms in an expansion represent products of certain input variables and their complements. If the presence of a certain input variable is assigned the designation 0, the presence of its complement the designation 1, and the absence of the variable the designation 2, we see that a term in an expansion such as $x^5\ \bar{x}^3\ x^1$, would correspond to the set of numbers 0 2 1 2 0, if we are dealing with five input variables.

Such a set of numbers corresponds to a number in the ternary number system which may be used to uniquely define the term $x^5\ \bar{x}^3\ x^1$. A polynomial expansion, consisting of a series of such terms is represented in the machine by

LOCATION	ORDER	NOTES Q1
18	LL 4095F	p-1
	LL 4095S3	
19	40 S4	
	22 64L	Store in accumulator
20	00 146F	
	49 584F	1st gate
21	00 224F	
	70 896F	2nd gate
22	00 255F	
	80 F	3rd gate
23	00 127F	
	LL 4095F	Complementor
24	50 F	
	S5 20F	Transfer word
25	50 F	
	22 F	Selector word
26	50 11L	
	10 37F	Select type
27	L4 71L	of algebra
	42 73L	
28	01 1F	Construct complement test
	40 F	
29	L1 45L	Go to direct algebra
	S4 74L	
30	32 83L	
	01 2F	Store M_1 in 1F
31	40 1F	
	01 2F	
32	L0 12L	Store M_2 - 3
	40 2F	
33	09 1F	Has c been stepped?
	L4 16L	
34	36 52L	
	L4 11L	Step c
35	40 16L	
	L5 12L	

Q1

LOCATION	ORDER	NOTES
36	LO 16L	Is c > 3?
	32 41L	
37	L5 29L	Transfer back to shift
	42 54L	
38	L5 16L	k = c
	42 46L	
39	L4 7L	
	00 20F	
40	46 101L	Store gate address
	L5 14L	Transfer Q
41	26 46L	
	L5 59L	Transfer to select
42	42 54L	
	L5 16L	
43	LO 12L	k = c - 3
	42 46L	
44	51 14L	
	00 1F	Transfer Q/27
45	66 13L	
	01 38F	
46	40 15L	
	19 F	
47	50 31L	
	42 17L	3^{k-1} in 17L
48	00 1F	
	L4 17L	
49	32 47L	
	51 15L	
50	00 1F	
	66 17L	q in 15L
51	10 1F	
	26 227L	
52	L5 15L	
	L4 2F	
53	36 54L	M_2 + q (mod 3) in 2F
	L4 12L	

LOCATION	ORDER	NOTES Q 1
54	40 2F	
	26 F	Go to shift or select
55	S5 156L	
	26 1F	Direct
56	46 99L	
	26 99L	and
57	46 62L	or
	22 61L	
58	46 59L	
	46 60L	+
59	L5 F	
	S4 85L	
60	J0 F	
	S0 92L	
61	22 63L	
	46 63L	
62	L5 F	or
	S4 98L	
63	J0 F	
	S0 100L	
64	26 1F	
	L5 2F	Step selector
65	L4 4L	
	46 2F	
66	L5 1F	
	L4 51L	Step accumulator
67	46 1F	
	L0 91L	
68	00 9F	
	10 63F	Delay
69	32 123L	End of order
	26 2F	Go to select
70	L1 F	
	36 73L	Complement test
71	L5 23L	
	S0 55L	Complement

Q1

LOCATION	ORDER	NOTES
72	40 3F	
	51 3F	
73	L5 1F	
	26 F	Go to algebra
74	L5 63L	
	42 25L	Return to shift
75	51 17L	
	75 1F	
76	L5 11L	$M_1 \times 3^{k-1} + 1$
	S4 12F	
77	42 101L	
	51 17L	
78	75 2F	$M_2 \times 3^{k-1} + 1$
	L5 11L	
79	S4 F	
	42 100L	
80	L5 24L	Plant select
	46 25L	
81	L5 25L	
	40 2F	
82	L5 19L	Plant store
	40 1F	
83	26 2F	Select
	L5 62L	Direct algebra
84	42 25L	
	26 80L	
85	L5 60L	
	42 25L	
86	51 17L	
	75 1F	$- 3^{k-1} M_1$
87	S1 156L	
	40 4F	
88	50 17L	
	74 2F	$(M_2 - M_1) 3^{k-1}$
89	S4 103L	
	00 20F	+ address

Q 1

LOCATION	ORDER	NOTES
90	L4 24L	
	22 80L	
91	S5 F	
	L4 24L	Waste
92	22 80L	
	41 3F	
93	L5 4F	
	32 94L	Did we want selection?
94	50 3F	
	L4 11L	
95	40 4F	
	L0 17L	
96	36 97L	Have we stepped too far?
	22 98L	
97	L0 17L	
	L0 17L	Return
98	40 4F	
	26 70L	
99	J0 F	
	S5 S4	and
100	26 1F	
	00 F	shift
101	J0 F	
	11 F	
102	26 70L	
	0F F	
103	L5 18L	
	L0 12L	Interlude
104	36 105L	
	L1 11L	forms
105	L4 30L	
	42 106L	$3^{p-3} + S4 - 1$
106	41 F	if p 3
	19 F	
107	50 31L	and S4
	42 F	otherwise

Q 1

LOCATION	ORDER	NOTES
108	00 1F	
	L4 F	
109	32 107L	
	L5 F	
110	00 20F	
	L4 19L	
111	L0 9L	
	46 91L	
112	L5 113L	
	22 1014F	
113	00 F	
	26 103L	
	26 1N	
103	L5 134L	
	22 125L	Store
104	L5 24L	
	26 145L	Escape
105	L5 144L	Transfer control
	26 136L	
106	10 20F	
	01 4F	Transfer p-1 times
107	L0 18L	
	32 117L	Test for disobey
108	L5 2L	
	46 112L	Locate order
109	10 20F	
	42 112L	
110	32 111L	
	19 26F	Right
111	26 112L	
	19 6F	or left
112	L4 F	
	40 F	Step order
113	L5 16L	
	00 1F	Restore c

Q1

LOCATION	ORDER	NOTES
114	10 1F	
	40 16L	
115	L5 24L	
	40 2L	Go out
116	S5 181L	
	32 123L	
117	26 2L	
	41 16L	Disobey and clear c
118	50 18L	
	L5 2L	
119	46 122L	
	46 123L	
120	36 121L	left
	22 121L	or right
121	00 20F	
	00 12F	
122	L5 F	
	S0 F	Restore order
123	40 F	
	L5 2L	
124	36 1L	Go out
	L4 10L	
125	22 1L	
	40 1F	Plant S4
126	L5 24L	
	46 135L	Plant address
127	L5 135L	
	40 2F	
128	26 1F	
	L5 2F	
129	L4 4L	Step address
	46 2F	
130	L5 1F	
	L4 51L	
131	46 1F	
	10 91L	

LOCATION	ORDER	NOTES Q1
132	00 9F	
	10 63F	Delay end
133	32 123L	
	26 1F	
134	L5 S4	
	26 2F	Parameters
135	40 F	
	22 128L	
136	40 F	
	41 1F	
137	26 F	
	L4 1F	Add terms
138	40 1F	
	L5 F	Step S4
139	L4 4L	
	46 F	
140	L0 91L	
	00 9F	
141	10 63F	Delay
	32 142L	end
142	26 F	
	L5 1F	
143	32 123L	
	26 115L	Transfer control
144	L1 S4	parameter
	22 137L	
145	46 147L	
	10 20F	
146	42 147L	escape
	S5 F	
147	32 F	
	26 F	
148	L5 154L	
	40 F	
149	26 F	Clear accumulator
	L5 F	

Q 1

LOCATION	ORDER	NOTES
150	L4 4L	
	46 F	
151	LO 91L	
	00 9F	
152	10 63F	Delay
	00 63F	
153	32 F	Go out
	26 F	
154	41 S4	
	22 149L	Parameter
155	L5 55L	
	46 153L	Prepare to read
156	26 148L	
	41 F	
157	L4 99L	
	42 169L	
158	00 20F	
	46 169L	Prepare store
159	LO 91L	
	00 9F	
160	32 123L	
	41 F	
161	41 2F	
	81 8F	
162	00 32F	Read in digits
	40 3F	
163	36 170L	
	LJ F	
164	10 3F	Convert to ternary
	42 2F	
165	00 1F	
	L4 2F	
166	32 164L	
	L5 2F	
167	L4 76L	Plant shift
	42 168L	

Q1

LOCATION	ORDER	NOTES
168	32 168L	Waste
	19 F	
169	L4 F	Store digit
	40 F	
170	L3 3F	
	36 174L	
171	L5 F	
	L4 11L	Step term counter
172	40 F	
	L5 3F	
173	L4 3F	Go back
	22 162L	
174	LJ 1F	
	L4 11L	Step register counter
175	42 1F	
	10 5F	
176	42 2F	Convert to ternary
	00 1F	
177	L4 2F	
	36 176L	
178	L5 2F	
	26 157L	Go back
179	92 133F	
	92 961F	
180	L5 19L	
	46 183L	
181	19 7F	
	40 F	Number of terms per line
182	41 1F	
	41 2F	
183	L5 F	
	00 13F	First set of terms
184	40 3F	
	32 200L	
185	L5 18L	
	40 4F	p-1 in 4

Q 1

LOCATION	ORDER	NOTES
186	50 11L	
	75 12L	
187	L5 4F	
	L0 11L	3^p
188	40 4F	
	32 186L	
189	S5 212L	
	40 4F	
190	L5 1F	
	50 31L	Prepare to print
191	66 4F	
	L5 18L	
192	40 4F	
	75 12L	
193	00 36F	
	82 4F	Print term
194	10 40F	
	L5 4F	
195	L0 11L	
	40 4F	
196	32 192L	
	92 961F	
197	L5 F	
	L4 F	
198	36 200L	New line
	92 129F	
199	92 961F	
	19 7F	
200	40 F	
	L5 1F	
201	L4 11L	Step term
	40 1F	
202	L5 2F	
	L4 11L	Step number in register
203	40 2F	
	L0 13L	next 27 digits?

Q 1

LOCATION	ORDER	NOTES
204	32 206L	
	00 63F	delay
205	L5 3F	
	L4 3F	Go back
206	26 184L	
	L5 183L	
207	L4 4L	
	46 183L	Are we through?
208	L0 91L	
	00 9F	
209	36 210L	Yes
	22 182L	No
210	92 770F	
	22 123L	Go out
211	L5 189L	
	46 153L	Prepare to read
212	26 148L	
	L5 18L	Store p-1
213	40 F	
	00 40F	Clear R_2
214	S5 F	
	40 1F	
215	81 4F	Test for N
	L0 12L	
216	32 123L	
	50 1F	
217	L4 12L	Convert to ternary
	74 12L	
218	L5 F	
	L0 11L	
219	40 F	
	36 214L	
220	01 1F	
	66 13L	
221	00 19F	
	L4 223L	Plant in shift

Q1

LOCATION	ORDER	NOTES
222	46 225L	Plant in shift
	L5 19L	
223	S4 12F	Plant in add
	46 226L	and store
224	10 20F	
	42 225L	
225	19 F	
	L4 F	Insert term
226	40 F	
	22 212L	
227	10 1F	
	66 12L	
228	10 1F	Finish calculation of q.
	40 15L	
229	26 52L	
	00 F	